Problem 1: Generate four values from LogNor (n, 1.96). Where n is the result of adding all digits of your phone number divided by 5 and rounded up. Use RV-bank beginning with the closest value to 0.66, then moving column-wise.

Solution

My phone number is 443-885-4241 which results in (4+4+3+8+8+5+4+2+4+1)/5 = 43/5 = 8.6, rounded up to 9. So, n = 9. To generate random numbers for LogNor (9, 1.96) we need to first generate numbers from N (9, 1.96). I also need to find closest number to 0.66 in my RV Bank. A simple search gave me 0.662 (use Ctrl+F and enter 0.66 and find smallest value in your RV Bank that begins with 0.66, then repeat the same process for 0.65 and find the largest number that begin with 0.65 – of the two numbers whichever is closest, choose that). Moving column-wise, my U (0, 1) values are:

u1 = 0.662, u2 = 0.599, u3 = 0.122, u4 = 0.907, u5 = 0.628, u6 = 0.838, u7 = 0.680, u8 = 0.634 and more if needed.

9	0.656	0.329	0.307	0.982	0.917	0.131	0.309
10	0.779	0.642	0.192	0.888	0.769	0.687	0.003
11	0.751	0.713	0.036	0.036	0.662	0.291	0.861
12	0.631	0.142	0.775	0.668	0.599	0.081	0.831
13	0.116	0.233	0.039	0.960	0.122	0.036	0.578
14	0.385	0.607	0.202	0.855	0.907	0.496	0.856
15	0.534	0.449	0.853	0.091	0.628	0.624	0.447
16	0.979	0.763	0.530	0.809	0.838	0.321	0.058
17	0.782	0.037	0.419	0.352	0.680	0.673	0.873
18	0.862	0.679	0.754	0.547	0.634	0.331	0.578
19	0.284	0.816	0.641	0.526	0.348	0.428	0.874
20	0.504	0.100	0.678	0.966	0.527	0.564	0.747

Variance is 1.96, yielding the value of standard deviation as 1.4 and following the steps of the algorithm:

$\mu = 9$, $\sigma = 1.4$

$u_1 = 0.662$, $u_2 = 0.599$,

$V_1^2 = 2\ u_1 - 1 = 2(0.662) - 1 = 0.324$, $V_2 = 2\ u_2 - 1 = 2\ (0.599) - 1 = 0.198$

$W = V_1^2 + V_2^2 = (0.324)^2 + (0.198)^2 = 0.1442 < 1$

$$y = \sqrt{\frac{-2\ ln\ w}{w}} = [(-2Ln\ 0.1442)/0.1442]^{0.5} = 5.1831$$

$Z_1 = V_1Y = (0.324)\ (5.1831) = 1.67934$ and $Z_2 = V_2Y = (0.198)\ (5.1831) = 1.02626$

These values are from N (0, 1) to convert them to N (9, 1.96) we need to use the equation $X = \sigma Z + \mu$

$XN_1 = (1.4)\ (1.67934) + 9 = 11.351$ and $XN_2 = (1.4)\ (1.02626) + 9 = 10.4368$.

In addition, to find the values for LogNor (9, 1.96) we need to perform:

$XLN_1 = e^{XN1} = e^{11.351} = 85056.57$ and $XLN_2 = e^{XN2} = e^{10.4368} = 34090.23$.

The next two numbers are created the same way. Here are the calculations:

$\mu = 9$, $\sigma = 1.4$

$u_1 = 0.122$, $u_2 = 0.907$

$V_1^2 = 2\,u_1 - 1 = -0.756$, $V_2 = 2\,u_2 - 1 = 0.814$

$W = V_1^2 + V_2^2 = 1.23413 > 1$

Since W is greater than 1, we need to go back to step 2 and generate two new U (0, 1) values.

$u_1 = 0.628$, $u_2 = 0.838$

$V_1^2 = 2\,u_1 - 1 = 0.256$, $V_2 = 2\,u_2 - 1 = 0.676$

$W = V_1^2 + V_2^2 = 0.5225 < 1$

$Y = 1.5762$

$Z_1 = 0.4035$ and $Z_2 = 1.0655$

$XN_1 = 9.565$ and $XN_2 = 10.492$.

$XLN_1 = 14255.93$ and $XLN_2 = 36017.6$.

Normal and Lognormal (μ, σ^2) Distributions

r.v.	μ	σ^2	σ					
	9	1.96	1.4					
0.662								
0.599		V_1	W	W<=1?	y	z1	XN1	XLN1
		0.324	0.14418	YES	5.183139	1.679337	11.35107	85056.57
		V_2				z2	XN2	XLN2
		0.198				1.026262	10.43677	34090.23

Normal and Lognormal (μ, σ^2) Distributions

r.v.	μ	σ^2	σ					
	9	1.96	1.4					
0.122								
0.907		V_1	W	W<=1?	y	z1	XN1	XLN1
		-0.756	1.234132	NO	NO	NO	NO	NO
		V_2				z2	XN2	XLN2
		0.814				NO	NO	NO

Normal and Lognormal (μ, σ^2) Distributions

r.v.	μ	σ^2	σ					
	9	1.96	1.4					
0.628								
0.838		V_1	W	W<=1?	y	z1	XN1	XLN1
		0.256	0.522512	YES	1.57625	0.40352	9.564928	14255.93
		V_2				z2	XN2	XLN2
		0.676				1.065545	10.49176	36017.6

Problem 2: Use your RV-Bank beginning from random number in row 8, column 8 and moving column-wise. Generate one random number from a gamma distribution where β = 2nd largest number in your cell phone number and α = 0.length of your longest finger in millimeters.

Solution

My phone number: 443-885-4241. Second largest number is 5, so, β = 5. Length of my longest finger 4.25 inches equivalent to 82.55 millimeters. So, α = 0.885.

Starting from the cell at (8, 8) and moving column-wise, here are the values of U (0, 1) to use:

u1 = 0.297, u2 = 0.462, u3 = 0.957, u4 = 0.627, u5 = 0.803, u6 = 0.995, u7 = 0.720, u8 = 0.446 and more if needed.

	1	2	3	4	5	6	7	8	9
1	0.834	0.531	0.123	0.780	0.424	0.278	0.740	0.802	0.99
2	0.468	0.558	0.909	0.835	0.956	0.199	0.113	0.149	0.74
3	0.024	0.354	0.860	0.649	0.094	0.664	0.190	0.770	0.35
4	0.896	0.783	0.545	0.281	0.823	0.781	0.548	0.466	0.02
5	0.846	0.829	0.696	0.581	0.718	0.587	0.032	0.111	0.70
6	0.796	0.740	0.506	0.487	0.255	0.773	0.330	0.286	0.45
7	0.182	0.675	0.449	0.025	0.381	0.652	0.058	0.118	0.52
8	0.120	0.924	0.951	0.246	0.867	0.983	0.903	0.297	0.44
9	0.656	0.329	0.307	0.982	0.917	0.131	0.309	0.462	0.79
10	0.779	0.642	0.192	0.888	0.769	0.687	0.003	0.957	0.86
11	0.751	0.713	0.036	0.036	0.662	0.291	0.861	0.627	0.40
12	0.631	0.142	0.775	0.668	0.599	0.081	0.831	0.803	0.58
13	0.116	0.233	0.039	0.960	0.122	0.036	0.578	0.995	0.63
14	0.385	0.607	0.202	0.855	0.907	0.496	0.856	0.720	0.04
15	0.534	0.449	0.853	0.091	0.628	0.624	0.447	0.446	0.50
16	0.979	0.763	0.530	0.809	0.838	0.321	0.058	0.279	0.15

Using the algorithm provided in the handout:

$\alpha = 0.885$, $\beta = 5$

$b = (e + \alpha)/e = 1.3255$

$u1 = 0.297$, $P = b\ u1 = 0.3937 < 1$

$y = P^{(1/\alpha)} = 0.3488$, $u2 = 0.462 < e^{-y} = 0.706$, therefore, $w = y = 0.3488$

This value is from Gamma (0.885, 1). In order to convert it to Gamma (0.885, 5), simply multiply it by 5.

X = 1.744.

Problem 3: Use your RV-Bank beginning from random number in row 4, column 4 and moving row-wise. Generate one random number from a gamma distribution where β = 2nd largest number in your student ID number and α = month of your birth.length of your shortest finger in millimeters.

	1	2	3	4	5	6	7	8	9	10	11
1	0.834	0.531	0.123	0.780	0.424	0.278	0.740	0.802	0.998	0.944	0.323
2	0.468	0.558	0.909	0.835	0.956	0.199	0.113	0.149	0.746	0.405	0.828
3	0.024	0.354	0.860	0.649	0.094	0.664	0.190	0.770	0.350	0.970	0.990
4	0.896	0.783	0.545	0.281	0.823	0.781	0.548	0.466	0.026	0.314	0.997
5	0.846	0.829	0.696	0.581	0.718	0.587	0.032	0.111	0.703	0.111	0.855
6	0.796	0.740	0.506	0.487	0.255	0.773	0.330	0.286	0.453	0.931	0.728
7	0.182	0.675	0.449	0.025	0.381	0.652	0.058	0.118	0.523	0.922	0.227
8	0.120	0.924	0.951	0.246	0.867	0.983	0.903	0.297	0.440	0.075	0.383

u1 = 0.281, u2 = 0.823, u3 = 0.781, u4 = 0.548, u5 = 0.466, u6 = 0.026, u7 = 0.314 and more if needed.

Let us assume that the birth month is March and length of shortest finger is 2.125 inches, which is equivalent to 54 millimeters and second largest number in student ID is 7. Then, α = 3.54, and β = 7.

Using the algorithm provided in the handout:

$$a = \frac{1}{\sqrt{2\alpha - 1}}, \quad b = \alpha - \ln 4, \quad q = \alpha + \frac{1}{a}, \quad \theta = 4.5, \quad d = 1 + \ln \theta$$

a = 0.4055, b = 2.1537, q = 6.0057, θ = 4.5, and d = 2.5041.

u1 = 0.281, and u2 = 0.823.

$$v = a \ln\left[\frac{u_1}{1 - u_1}\right], \quad y = \alpha e^{v}, \quad z = u_1^2 u_2, \text{ and } w = b + qv - y$$

v = - 0.3810, y = 2.4184, z = 0.065, w = -2.553

Since $w + d - \theta z$ = - 0.3413 < 0 , we will proceed to step 5.

Since w = -2.553 > Ln z = - 2.7336, then T = y = 2.4184.

This value is from Gamma (3.54, 1). In order to convert it to Gamma (3.54, 7), simply multiply it by 7.

X = 16.9288

Gamma (α, β) Distribution for α >1								
r.v.	α	β	a	b	q	θ	d	
	3.54	7	0.405554	2.153706	6.005766	4.5	2.504077	
0.281								
0.823		v	y	z	w	w + d - θz	T	X
		-0.38102	2.418401	0.064985	-2.55301	-0.34137	NO	NO
						ln z	T	X
						-2.7336	2.418401	16.9288

Problem 4: Use your RV-Bank beginning from random number in row 20, column 20 and moving column-wise. Generate five random numbers from Tria (4.11, 6.nn, 8.74), where nn is two digits of your birth day.

Solution

Let us assume, nn = 15, TRIA (4.11, 6.15, 8.74). u1 =0.268 (from cell (20, 20)), u2 = 0.834 (from cell (1, 1)), u3 = 0.468 (from cell (2, 1)), u4 = 0.024 (from cell (3, 1)), and u5 = 0.896 (from cell (4, 1)).

$$k = \frac{m - a}{b - a}$$

K = (6.15 – 4.11) / (8.74 – 4.11) = 0.441

u1 =0.268 < 0.441, then $y = \sqrt{k\,u}$ = 0.344, $x = a + (b - a)y$, X = 5.263

u2 = 0.834 > 0.441, then $y = 1 - \sqrt{(1\text{-}k)(1\text{-}u)}$ = 0.086, X = 5.688

u3 = 0.468 > 0.441, then $y = 1 - \sqrt{(1\text{-}k)(1\text{-}u)}$ = 0.316, X = 5.126

u4 = 0.024 < 0.441, then $y = \sqrt{k\,u}$ = 0.103

u5 = 0.896 > 0.441, then $y = 1 - \sqrt{(1\text{-}k)(1\text{-}u)}$ = 0.053, X = 5.11