1. Use Microsoft Excel. Generate 400 random numbers from U (0, 1), organized in 20 rows and 20 columns, only 3 digits after the decimal.

You will be using this set in later assignments, so make sure to save a copy for your future use.

For future reference, we call this set your RV Bank so when you are asked to use your RV Bank you know where to look for the information. Send the Excel file to me separate from your homework (remember to use correct naming format). All references to elements of the RV Bank are in matrix format.

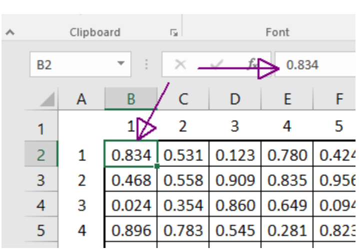
For example, when you need to start from (12,4), the reference is to the value at 12th row and 4th column.

Every time that you use random numbers from your RV Bank for a problem, you need to have a small image of the RV Bank with selected values highlighted

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	0.834	0.531	0.123	0.780	0.424	0.278	0.740	0.802	0.998	0.944	0.323	0.980	0.968	0.678	0.704	0.391	0.530	0.294	0.788	0.639
2	0.468	0.558	0.909	0.835	0.956	0.199	0.113	0.149	0.746	0.405	0.828	0.748	0.788	0.103	0.764	0.395	0.861	0.267	0.239	0.348
3	0.024	0.354	0.860	0.649	0.094	0.664	0.190	0.770	0.350	0.970	0.990	0.537	0.241	0.142	0.821	0.679	0.325	0.443	0.393	0.572
4	0.896	0.783	0.545	0.281	0.823	0.781	0.548	0.466	0.026	0.314	0.997	0.928	0.813	0.131	0.541	0.452	0.288	0.868	0.439	0.061
5	0.846	0.829	0.696	0.581	0.718	0.587	0.032	0.111	0.703	0.111	0.855	0.257	0.272	0.192	0.799	0.692	0.771	0.044	0.842	0.832
6	0.796	0.740	0.506	0.487	0.255	0.773	0.330	0.286	0.453	0.931	0.728	0.989	0.163	0.608	0.350	0.307	0.092	0.210	0.341	0.021
7	0.182	0.675	0.449	0.025	0.381	0.652	0.058	0.118	0.523	0.922	0.227	0.158	0.067	0.540	0.019	0.477	0.450	0.529	0.439	0.796
8	0.120	0.924	0.951	0.246	0.867	0.983	0.903	0.297	0.440	0.075	0.383	0.562	0.888	0.000	0.352	0.290	0.751	0.702	0.709	0.116
9	0.656	0.329	0.307	0.982	0.917	0.131	0.309	0.462	0.790	0.423	0.560	0.017	0.035	0.006	0.373	0.005	0.257	0.788	0.172	0.612
10	0.779	0.642	0.192	0.888	0.769	0.687	0.003	0.957	0.862	0.572	0.875	0.603	0.832	0.685	0.019	0.413	0.468	0.375	0.656	0.437
11	0.751	0.713	0.036	0.036	0.662	0.291	0.861	0.627	0.400	0.819	0.874	0.081	0.476	0.196	0.079	0.009	0.633	0.948	0.581	0.591
12	0.631	0.142	0.775	0.668	0.599	0.081	0.831	0.803	0.580	0.476	0.725	0.086	0.791	0.432	0.097	0.010	0.671	0.951	0.729	0.710
13	0.116	0.233	0.039	0.960	0.122	0.036	0.578	0.995	0.635	0.377	0.338	0.577	0.629	0.465	0.771	0.729	0.718	0.000	0.733	0.470
14	0.385	0.607	0.202	0.855	0.907	0.496	0.856	0.720	0.047	0.125	0.690	0.219	0.182	0.719	0.074	0.284	0.593	0.798	0.911	0.587
15	0.534	0.449	0.853	0.091	0.628	0.624	0.447	0.446	0.507	0.190	0.553	0.225	0.932	0.394	0.897	0.738	0.322	0.592	0.782	0.485
16	0.979	0.763	0.530	0.809	0.838	0.321	0.058	0.279	0.152	0.192	0.814	0.706	0.495	0.236	0.536	0.070	0.644	0.942	0.596	0.912
17	0.782	0.037	0.419	0.352	0.680	0.673	0.873	0.728	0.518	0.336	0.484	0.500	0.025	0.614	0.259	0.261	0.190	0.535	0.164	0.329
18	0.862	0.679	0.754	0.547	0.634	0.331	0.578	0.569	0.575	0.074	0.152	0.195	0.122	0.794	0.981	0.428	0.550	0.768	0.458	0.549
19	0.284	0.816	0.641	0.526	0.348	0.428	0.874	0.044	0.222	0.371	0.193	0.961	0.972	0.911	0.339	0.216	0.012	0.231	0.615	0.584
20	0.504	0.100	0.678	0.966	0.527	0.564	0.747	0.357	0.959	0.680	0.625	0.916	0.522	0.115	0.955	0.982	0.954	0.364	0.286	0.268

Explanation of Solution

I used Excel and RAND function to create the 400 random numbers in 20 rows and 20 columns as instructed. I highlighted the entire set and changed the number of decimals to three then I copied the entire set and pasted them to Notepad. In Notepad, I again copied the entire set and copied them back to Excel sheet. I made sure that the numbers were actually fixed numbers with three digits of decimal.



Then I saved the sheet with correct naming format, MS_IEGR_MyRVBank. Then I sent my Excel sheet to the instructor and it was verified as correct.

2. An experiment involves throwing a die three times and recording the total number of dots observed. Generate eight random numbers simulating this experiment using the first eight random numbers generated in problem 1.

Solution

Since no row or column instructions are given, I will select the first eight numbers in row 1.

	1	2	3	4	5	6	7	8	9
1	0.834	0.531	0.123	0.780	0.424	0.278	0.740	0.802	0.998
2	0.468	0.558	0.909	0.835	0.956	0.199	0.113	0.149	0.746
3	0.024	0.354	0.860	0.649	0.094	0.664	0.190	0.770	0.350
4	0.896	0.783	0.545	0.281	0.823	0.781	0.548	0.466	0.026
5	0.846	0.829	0.696	0.581	0.718	0.587	0.032	0.111	0.703

u1 = 0.834, u2 = 0.531, u3 = 0.123, u4 = 0.780, u5 = 0.424, u6 = 0.278, u7 = 0.740, u8 = 0.802

To find out what the distribution is we should find all possible scenarios and count them. Because we are counting only the total numbers, we do not need to distinguish between the throws. For example, if we are counting all cases that might result in a total number of dots to be eight, then here are the possibilities

(1+1+6), (1+2+5), (1+3+4), (2+2+4), (2+3+3) 4 cases

Note that we are not distinguishing between dice. For example in the above (1+3+4) does not mean the first throw is one, the second throw 3 and the third throw. It simply means one of the throws is 1, another one 3, and another one 4. No order is assumed. 1 might occur in the firs throw, in the second throw or the third throw. Now let us count the cases. The minimum is 3 (when you have three 1s) and the maximum is 18 (when you have three 6s). There is only one possibility for each. What about the total of 4? Again, there is only one case (1+1+2). What about the total of 17? Again only one case, (6+6+5). Let us do one more, the total of 5. There will be two possibilities: (1+1+3) and (1+2+2). Not surprisingly for the total of 16 we will only have two possibilities: (6+6+4) and (6+5+5). This was done to show you that this discrete distribution is actually symmetrical which means the counts for 3, 4, 5, 6, 7, 8, and 9 are exactly as the counts for 16, 15, 14, 13, 12, 11, and 10, respectively. Here are the counts:

X=3 → 1 X=18 → 1

X=4 → 1 X=17 → 1

X=5 → 2 X=16 → 2

X=6 → 3 X=15 → 3

X=7 → 4 X=14 → 4

X=8 → 5 X=13 → 5

X=9 → 6 X=12 → 6

X=10 → 7 X=11 → 7

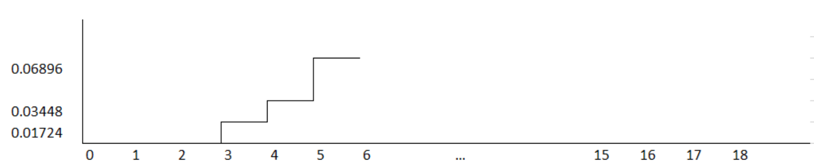
Total possibilities: (2)(1+1+2+3+4+5+6+7) = 2(29) = 58. Therefore, probability of getting the three dots is 1/58, or 0.017241379, which is similar to getting total dots of 18. Thus, $P_X(x)$ is written as:

P(X)	0.01724	X = 3
	0.01724	X = 4
	0.03448	X = 5
	0.05172	X = 6
	0.06897	X = 7
	0.08621	X = 8
	0.10345	X = 9
	0.12069	X = 10
	0.12069	X = 11
	0.10345	X = 12
	0.08621	X = 13
	0.06897	X = 14
	0.05172	X = 15
	0.03448	X = 16
	0.01724	X = 17
	0.01724	X = 18

We now need to calculate $F_X(x)$ and plot it.

F(X)	0	X < 3
	0.01724	X <= 3
	0.03448	X <= 4
	0.06897	X <= 5
	0.12069	X <= 6
	0.18966	X <= 7
	0.27586	X <= 8
	0.37931	X <= 9
	0.5	X <= 10
	0.62069	X <= 11
	0.72414	X <= 12
	0.81034	X <= 13
	0.87931	X <= 14
	0.93103	X <= 15
	0.96552	X <= 16
	0.98276	X <= 17
	1	X <= 18
	1	X > 18

Partial plot is illustrated here.



For each one of the random values we can use the plot of $F_X(x)$ or the calculated values and find the proper X value. For example for u1 = 0.834 we can see that it is between 0.8103 and 0.8793, therefore X=14 and so on.

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3. Add all digits of your phone number. Now, divide it by 3. Use that as a decimal value between 0 and 1 to represent the probability of success in a Bernoulli experiment. For example, a phone number of 443-885-4241 will be 4+4+3+8+8+5+4+2+4+1 = 43 and third of it will be 14.3 making p=0.143. Use Excel to generate 100 Bernoulli (p) and 100 Geo (p) using your RV-bank beginning from a cell that has closest value to 0.5 and moving column-wise. Use the generated Bernouli values to generate 8 values from Bin (10, p). Use the generated Geometric values to generate 8 values from NegBin (10, p).

Solution

My phone number: 443-885-4241 which results in (4+4+3+8+8+5+4+2+4+1)/3 = 14.33, so p = 0.1433.

In my RV Bank I have two numbers closest to 0.5 with equal distance. So I will select one of them arbitrary. I have selected 0.504. To generate 100 random variables from Bernoulli, I need 100 numbers from U (0, 1). The same is true for Geometric distribution. I will select 200 values from my RV Bank.

	1	2	3	4	5	6	7	8	9	10	11
1	0.834	0.531	0.123	0.780	0.424	0.278	0.740	0.802	0.998	0.944	0.323
2	0.468	0.558	0.909	0.835	0.956	0.199	0.113	0.149	0.746	0.405	0.828
3	0.024	0.354	0.860	0.649	0.094	0.664	0.190	0.770	0.350	0.970	0.990
4	0.896	0.783	0.545	0.281	0.823	0.781	0.548	0.466	0.026	0.314	0.997
5	0.846	0.829	0.696	0.581	0.718	0.587	0.032	0.111	0.703	0.111	0.855
6	0.796	0.740	0.506	0.487	0.255	0.773	0.330	0.286	0.453	0.931	0.728
7	0.182	0.675	0.449	0.025	0.381	0.652	0.058	0.118	0.523	0.922	0.227
8	0.120	0.924	0.951	0.246	0.867	0.983	0.903	0.297	0.440	0.075	0.383
9	0.656	0.329	0.307	0.982	0.917	0.131	0.309	0.462	0.790	0.423	0.560
10	0.779	0.642	0.192	0.888	0.769	0.687	0.003	0.957	0.862	0.572	0.875
11	0.751	0.713	0.036	0.036	0.662	0.291	0.861	0.627	0.400	0.819	0.874
12	0.631	0.142	0.775	0.668	0.599	0.081	0.831	0.803	0.580	0.476	0.725
13	0.116	0.233	0.039	0.960	0.122	0.036	0.578	0.995	0.635	0.377	0.338
14	0.385	0.607	0.202	0.855	0.907	0.496	0.856	0.720	0.047	0.125	0.690
15	0.534	0.449	0.853	0.091	0.628	0.624	0.447	0.446	0.507	0.190	0.553
16	0.979	0.763	0.530	0.809	0.838	0.321	0.058	0.279	0.152	0.192	0.814
17	0.782	0.037	0.419	0.352	0.680	0.673	0.873	0.728	0.518	0.336	0.484
18	0.862	0.679	0.754	0.547	0.634	0.331	0.578	0.569	0.575	0.074	0.152
19	0.284	0.816	0.641	0.526	0.348	0.428	0.874	0.044	0.222	0.371	0.193
20	0.504	0.100	0.678	0.966	0.527	0.564	0.747	0.357	0.959	0.680	0.625

After copying the values in a new sheet in Excel, I have calculated Ber (0.1433) values. Since probability of success is small, less than 15%, I should expect to get many zero values. I used an IF statement based on instructions from handout. If the U (0, 1) value was less than or equal to p=0.1433, then the Ber (0.1433) is assigned a value of 1, otherwise a value of 0. The image below shows how the values are calculate. For u = 0.504 which is not less than or equal to 0.1433 a value of 0 is assign for Ber (0.1433) while for u = 0.142 which is less than 0,1433 a value of 1 is assigned to Ber (0.1433). Similar approach is used using the procedure to generate random numbers from Bin (10, 0.1433). Here every 10 generated Ber (0.1433) were added to get one value of Bin (10, 0.1433).

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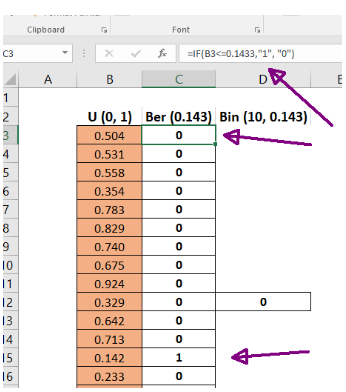
U (0, 1)	Ber (0.143)	Bin (10, 0.143)
0.504	0	
0.531	0	
0.558	0	
0.354	0	
0.783	0	
0.829	0	
0.740	0	
0.675	0	
0.924	0	
0.329	0	0
0.642	0	
0.713	0	
0.142	1	
0.233	0	
0.607	0	
0.449	0	
0.763	0	
0.037	1	
0.679	0	
0.816	0	2
0.100	1	
0.123	1	
0.909	0	
0.860	0	
0.545	0	
0.696	0	
0.506	0	
0.449	0	
0.951	0	
0.307	0	2
0.192	0	
0.036	1	
0.775	0	
0.039	1	
0.202	0	
0.853	0	
0.530	0	
0.419	0	
0.754	0	
0.641	0	2
0.678	0	
0.780	0	
0.835	0	
0.649	0	
0.281	0	
0.581	0	
0.487	0	
0.025	1	
0.246	0	
0.982	0	1

U (0, 1)	Ber (0.143)	Bin (10, 0.143)
0.888	0	
0.036	1	
0.668	0	
0.960	0	
0.855	0	
0.091	1	
0.809	0	
0.352	0	
0.547	0	
0.526	0	2
0.966	0	
0.424	0	
0.956	0	
0.094	1	
0.823	0	
0.718	0	
0.255	0	
0.381	0	
0.867	0	
0.917	0	1
0.769	0	
0.662	0	
0.599	0	
0.122	1	
0.907	0	
0.628	0	
0.838	0	
0.680	0	
0.634	0	
0.348	0	1
0.527	0	
0.278	0	
0.199	0	
0.664	0	
0.781	0	
0.587	0	
0.773	0	
0.652	0	
0.983	0	
0.131	1	
0.687	0	
0.291	0	
0.081	1	
0.036	1	
0.496	0	
0.624	0	
0.321	0	
0.673	0	
0.331	0	
0.428	0	



For the Geo (0.1433), according to the procedure in the handout we need to calculate the ratio,

LN (u(0, 1) / LN (1 – 0.1433) and the round it down. The formula in Excel was entered as

=INT(LOG(K8)/LOG(1-0.433))

Again for the NegBin (10, 0.1433), 10 generated numbers were used to generate one value of NegBin.

U (0, 1)	Geo (0.1433)	NegBin (10, 0.143)
0.564	1	
0.740	0	
0.113	3	
0.190	2	
0.548	1	
0.032	6	
0.330	1	
0.058	5	
0.903	0	
0.309	2	21
0.003	10	
0.861	0	
0.831	0	
0.578	0	
0.856	0	
0.447	1	
0.058	5	
0.873	0	
0.578	0	
0.874	0	16
0.747	0	
0.802	0	
0.149	3	
0.770	0	
0.466	1	
0.111	3	
0.286	2	
0.118	3	
0.297	2	
0.462	1	15
0.957	0	
0.627	0	
0.803	0	
0.995	0	
0.720	0	
0.446	1	
0.279	2	
0.728	0	
0.569	0	
0.044	5	8
0.357	1	
0.998	0	
0.746	0	
0.350	1	
0.026	6	
0.703	0	
0.453	1	
0.523	1	
0.440	1	
0.790	0	11

U (0, 1)	Geo (0.1433)	NegBin (10, 0.143)
0.862	0	
0.400	1	
0.580	0	
0.635	0	
0.047	5	
0.507	1	
0.152	3	
0.518	1	
0.575	0	
0.222	2	13
0.959	0	
0.944	0	
0.405	1	
0.970	0	
0.314	2	
0.111	3	
0.931	0	
0.922	0	
0.075	4	
0.423	1	11
0.572	0	
0.819	0	
0.476	1	
0.377	1	
0.125	3	
0.190	2	
0.192	2	
0.336	1	
0.074	4	
0.371	1	15
0.680	0	
0.323	1	
0.828	0	
0.990	0	
0.997	0	
0.855	0	
0.728	0	
0.227	2	
0.383	1	
0.560	1	
0.875	0	
0.874	0	
0.725	0	
0.338	1	
0.690	0	
0.553	1	
0.814	0	
0.484	1	
0.152	3	
0.193	2	

Problem 4. Generate two values from Poi (6.nn) where nn is the total values of phone number digits you calculated in Problem 3. Use RV-bank beginning with the closest value to the first two digits of your phone number in decimal format (e.g. 44, then 0.44) moving in row or column.

Solution:

Calculating nn → 4+4+3+8+8+5+4+2+4+1 = 43, So POI (6.43) is the target distribution.

First two digits of phone number is 44, so I will be looking for closest number to 0.44 in my RV Bank. I actually have a 0.44 in my RV Bank in row 8, column 9. I select the column numbers.

u1=0.440, u2=0.790, u3=0.862, u4=0.400, u5=0.580, u6=0.635, u7=0.047, u8=0.507, etc.

.587	0.032	0.111	0.703	0.111	0.
.773	0.330	0.286	0.453	0.931	0.
.652	0.058	0.118	0.523	0.922	0.
.983	0.903	0.297	0.440	0.075	0.
.131	0.309	0.462	0.790	0.423	0.
.687	0.003	0.957	0.862	0.572	0.
.291	0.861	0.627	0.400	0.819	0.
.081	0.831	0.803	0.580	0.476	0.
.036	0.578	0.995	0.635	0.377	0.
.496	0.856	0.720	0.047	0.125	0.
.624	0.447	0.446	0.507	0.190	0.
.321	0.058	0.279	0.152	0.192	0.
.673	0.873	0.728	0.518	0.336	0.
.331	0.578	0.569	0.575	0.074	0.
.428	0.874	0.044	0.222	0.371	0.
.564	0.747	0.357	0.959	0.680	0.

Applying the procedure:

$a = e^{-\lambda} = e^{-6.43} = 0.002$ $b = 1$

i =0 u1 = 0.440 b = (1)(0.440) = 0.440 > 0.002

i=1 u2 = 0.790 b = (0.440)(0.790) = 0.348 > 0.002

i=2 u3 = 0.862 b = (0. 348)(0. 862) = 0.300 > 0.002

i=3 u4 = 0.400 b = (0.300)(0.400) = 0.120 > 0.002

i=4 u5 = 0.580 b = (0.120)(0.580) = 0.070 > 0.002

i=5 u6 = 0.635 b = (0.070)(0.635) = 0.044 > 0.002

i=6 u7 = 0.047 b = (0.044)(0.0.047) = 0.002 = 0.002 (not smaller than)

i=7 u8 = 0.507 b = (0.002)(0.507) = 0.001 < 0.002

Therefore, X = I = 7

Second number can be calculated the same way.