

Assignment # 2: Generating Random Numbers - 1.

Point: 50

1. Generate all possible random numbers by the LCG (103, 11, c,  $X_0$ ). For  $X_0$  use the day number of your birthday. For c value, add the digits of your birth year. You can use Excel for calculations and copy it into your report, but you must explain what the numbers are and how they are calculated. Identify the period of LCG and if it is not a full period identify the random numbers that are missing. Use only three digits of decimals. Find  $U_{240307}$  for this LCG without using Excel.

2. Repeat the same steps for problem 1 but assuming that you have a MLCG.

**General Grading Scheme**

Points are assigned to presence of following items:

1. Correct solution
2. Explanation of how the solution was generated and how the values are calculated with sample calculations
3. Extra research to add to the quality of report (bonus points)

Points are deducted for following items:

1. Late submission
2. Not following submission guidelines
3. Lack of explanation

### Solution

1. Assume that birthday is Feb. 5, 1998. Then  $X_0 = 5$  and  $c = 1+9+9+8 = 27$ .  $a = 11$  and  $m = 103$  are already provided. Therefore, the LCG is  $X_i = (11X_{i-1} + 27) \bmod 103$ .

An EXCEL sheet was used to do the calculation.  $X_i$  values were calculated using the recursive formula. MOD function of EXCEL was used to calculate the values.

Sample calculation: For  $i=5$ , we have the following information,

$X_4 = 51$ ,  $a = 11$ ,  $c=27$  and  $m=103$ . Thus  $X_5 = (11*51 + 27) \bmod 103 = 588 \bmod 103$ . There are five 103s in 588, so  $588 - 5*103 = 588 - 515 = 73$ .

The formula used in EXCEL is: `=MOD((I$47*E53+J$47),K$47)`.

A period of 102 was accomplished which is remarkable (only 1 less than a full period of 103).

To calculate  $U_{240307}$  you simply need to find out what  $240307 \bmod 102$  is. That is how many periods are in 240307 and whatever the answer of the mod function is that will be used to find the value for  $U_{240307}$ .  $240307 \bmod 102 = 97$ , so the 97th value of 0.74757 is the answer.

2. A Multiplicative Linear Congruential Generator (MLCG) is calculated the same as an LCG but with  $c = 0$ . MLCG period is 102 again. So the 97th value of 0.24272 is the answer for  $U_{240307}$ .

LCG		
i	$X_i$	$U_i = X_i / m$
0	5	
1	82	0.796117
2	2	0.019417
3	49	0.475728
4	51	0.495146
5	73	0.708738
6	6	0.058252
7	93	0.902913
8	20	0.194175
9	41	0.398058
10	66	0.640777
11	32	0.31068
12	70	0.679612
13	76	0.737864
14	39	0.378641
15	44	0.427184
16	99	0.961165
17	86	0.834951
18	46	0.446602
19	18	0.174757
20	19	0.184466
21	30	0.291262
22	48	0.466019

**PERIOD = 102**

5	11	27	103
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23	40	0.38835
24	55	0.533981
25	14	0.135922
26	78	0.757282
27	61	0.592233
28	80	0.776699
29	83	0.805825
30	13	0.126214
31	67	0.650485
32	43	0.417476
33	88	0.854369
34	68	0.660194
35	54	0.524272
36	3	0.029126
37	60	0.582524
38	69	0.669903
39	65	0.631068
40	21	0.203883
41	52	0.504854
42	84	0.815534
43	24	0.23301
44	85	0.825243
45	35	0.339806
46	0	0
47	27	0.262136
48	15	0.145631
49	89	0.864078
50	79	0.76699
51	72	0.699029
52	98	0.951456
53	75	0.728155
54	28	0.271845
55	26	0.252427
56	4	0.038835
57	71	0.68932
58	87	0.84466
59	57	0.553398
60	36	0.349515
61	11	0.106796
62	45	0.436893
63	7	0.067961
64	1	0.009709
65	38	0.368932
66	33	0.320388
67	81	0.786408
68	94	0.912621
69	31	0.300971

**PERIOD = 102**

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70	59	0.572816
71	58	0.563107
72	47	0.456311
73	29	0.281553
74	37	0.359223
75	22	0.213592
76	63	0.61165
77	102	0.990291
78	16	0.15534
79	100	0.970874
80	97	0.941748
81	64	0.621359
82	10	0.097087
83	34	0.330097
84	92	0.893204
85	9	0.087379
86	23	0.223301
87	74	0.718447
88	17	0.165049
89	8	0.07767
90	12	0.116505
91	56	0.543689
92	25	0.242718
93	96	0.932039
94	53	0.514563
95	95	0.92233
96	42	0.407767
97	77	0.747573
98	50	0.485437
99	62	0.601942
100	91	0.883495
101	101	0.980583
102	5	0.048544
103	82	0.796117
104	2	0.019417
105	49	0.475728

**PERIOD = 102**

LCG		
i	$X_i$	$U_i = X_i / m$
0	5	
1	55	0.533981
2	90	0.873786
3	63	0.61165
4	75	0.728155
5	1	0.009709
6	11	0.106796
7	18	0.174757
8	95	0.92233
9	15	0.145631
10	62	0.601942
11	64	0.621359
12	86	0.834951
13	19	0.184466
14	3	0.029126
15	33	0.320388
16	54	0.524272
17	79	0.76699
18	45	0.436893
19	83	0.805825
20	89	0.864078
21	52	0.504854
22	57	0.553398
23	9	0.087379
24	99	0.961165
25	59	0.572816
26	31	0.300971
27	32	0.31068
28	43	0.417476
29	61	0.592233
30	53	0.514563
31	68	0.660194
32	27	0.262136
33	91	0.883495
34	74	0.718447
35	93	0.902913
36	96	0.932039
37	26	0.252427
38	80	0.776699
39	56	0.543689
40	101	0.980583
41	81	0.786408
42	67	0.650485

5	11	0	103
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**PERIOD = 102**

**PERIOD = 102**

43	16	0.15534
44	73	0.708738
45	82	0.796117
46	78	0.757282
47	34	0.330097
48	65	0.631068
49	97	0.941748
50	37	0.359223
51	98	0.951456
52	48	0.466019
53	13	0.126214
54	40	0.38835
55	28	0.271845
56	102	0.990291
57	92	0.893204
58	85	0.825243
59	8	0.07767
60	88	0.854369
61	41	0.398058
62	39	0.378641
63	17	0.165049
64	84	0.815534
65	100	0.970874
66	70	0.679612
67	49	0.475728
68	24	0.23301
69	58	0.563107
70	20	0.194175
71	14	0.135922
72	51	0.495146
73	46	0.446602
74	94	0.912621
75	4	0.038835
76	44	0.427184
77	72	0.699029
78	71	0.68932
79	60	0.582524
80	42	0.407767
81	50	0.485437
82	35	0.339806
83	76	0.737864
84	12	0.116505
85	29	0.281553
86	10	0.097087
87	7	0.067961
88	77	0.747573
89	23	0.223301

**PERIOD = 102**

**102**

90	47	0.456311
91	2	0.019417
92	22	0.213592
93	36	0.349515
94	87	0.84466
95	30	0.291262
96	21	0.203883
97	25	0.242718
98	69	0.669903
99	38	0.368932
100	6	0.058252
101	66	0.640777
102	5	0.048544
103	55	0.533981
104	90	0.873786
105	63	0.61165

**PERIOD = 1**