IEGR 350: Engineering Economy
Fall 2015
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Test 1

100 Points (Time: 6:00 Minutes)

Instruction for all problems: Show your work. No round down or up, use 2 decimals for dollar values and 4 decimals for factors.

Question 1: (10+10+10+10 +10 points)
Use tables and interpolations.

Is this a profitable investment? Buy a production machine for $30,000, receive profit of $1,000 at the end of first year increasing geometrically 20% each year to year 10 and then decreasing arithmetically to year 20 with year 20 profit the same as year 3 profit. Salvage value of the unit is $5,000. Interest rate is 7.4% compounded bi-monthly.

First let's identify the payment values at year 3 and year 10. Beginning from year 1 to year 10, there are 9 periods. So, year 10 value following the geometric series will be $1,000 $(1+0.20)^9$ which is equal to $5,159.78. Similarly for year 3 we have two periods and the value will be $1,000 $(1+0.20)^2$ = $1,440.00.

From year 10 to year 20 there are 10 periods with beginning value at year 10 as $5,159.78 and year 10 as $1,440.00. To calculate G, subtract two values and divide by 10.

G = (5159.78 – 1440)/10 = 371.97

Before we can use find the present worth to judge whether this investment is profitable or not we need to address the effective interest rate issue. Nominal interest rate is 7.4% but it is compounded bi-monthly. So, 6 times during a year it is compounded.

$$i_{eff} = (1 + \frac{0.074}{6})^6 - 1 = 0.0763 \ \ or \ 7.63\%$$

Now, let's write the present worth formulas for the series. Note that payment at year 10 is part of both series and can only be considered once. Let's consider it as part of geometric series. So our arithmetic series begins at year 11 and ends at year 20 (10 periods).

$$PW1 = \$1{,}000\left(\frac{P}{A}, g, i, n\right) = \$1{,}000\left[\frac{1 - (\frac{1+g}{1+i})^n}{i - g}\right] = \$1{,}000\left[\frac{1 - (\frac{1+0.2}{1+0.0763})^{10}}{0.0763 - 0.2}\right]$$

$$PW1 = \$1{,}000\left[\frac{1 - 2.96758}{-0.1237}\right] = \$1{,}000\,(15.9060) = \$15{,}906$$

The second series is a combination of annuities (year 11 value=$5,159.78 - $371.97 = $4,787.80) and an arithmetic series beginning with the first G (G=-371.97) at year 11. So the combined present worth of that series (occurring in year 10 to be brought to year zero using a (P/F, 7.63%, 10) factor) is:

PW2 = [$4,787.80 (P/A, 7.63%, 10) - $371.97 (P/G, 7.63%, 10)] (P/F, 7.63%, 10)

We need to find the values of three required factors by interpolation as required.

(P/A, 7.63%, 10):

(P/A, 7%, 10) X	(P/A, 8%, 10) Y	Difference for 1% Δ = Y-X	(P/A, 7.63%, 10) X+0.63Δ
7.0236	6.7101	-0.3135	7.0236+0.63(-0.3135)=6.8261

(P/G, 7.63%, 10)

(P/G, 7%, 10) X	(P/G, 8%, 10) Y	Difference for 1% Δ = Y-X	(P/G, 7.63%, 10) X+0.63Δ
27.7156	25.9768	-1.7388	27.7156+0.63(-1.7388)=26.6201

(P/F, 7.63%, 10)

(P/F, 7%, 10) X	(P/F, 8%, 10) Y	Difference for 1% Δ = Y-X	(P/F, 7.63%, 10) X+0.63Δ
0.5083	0.4632	-0.0451	0.5083+0.63(-0.0451)=0.4799

PW2 = [$4,787.80 (6.8261) - $371.97 (26.6201)] (0.4799) = $10,932.18

And present worth of the series is:

PW = - $30,000 + $15,906.60 + $10,932.18 = - $3,161.22 (Not a good investment)

Grading Rubric:

- 10 points for interpolation
- 10 points for present worth of geometric series
- 20 points for present worth of arithmetic series
- 10 points for problem set up, explanation and final conclusion

Question 2: (20 points)
In cleaning out some files that were left behind by the engineer who preceded you in your current job, you found an old report that had a calculation for the present worth of certain maintenance costs for state highways. The report contained the following equation (with cost in $1 million):

$$12\{1 - [(1 + 0.03)/(1 + 0.06)]^{x}\}/(0.06 - 0.03) = 140$$

The value of x that was used in the calculation was illegible. What is its value?

Solution:

Multiply both sides by (0.06-0.03)

$$12\{1 - [(1 + 0.03)/(1 + 0.06)]^{x}\} = 140(0.06 - 0.03) = 140(0.03) = 4.2$$
$$12\{1 - [(1.03)/(1.06)]^{x}\} = 4.2$$
$$12\{1 - [(0.9717)]^{x}\} = 4.2$$

Multiply both sides by 1/12

$$1 - [(0.9717)]^{x}\} = 4.2/12 = 0.35$$
$$- [(0.9717)]^{x}\} = 0.35 - 1 = -0.65$$
$$[(0.9717)]^{x} = 0.65$$

Take LOG from both sides

$$\text{Log } 0.65 = x \log 0.9717$$
$$X = 15 \text{ years}$$

Question 3: (30 points)
On a student loan, you have received $10,000 twice a year on January 1st and August 1st of each year for the last 9 semesters. Rate is 8% compounded monthly. Upon graduation, you start paying back your loan. What should your monthly payments be to pay back your loan in the same amount of months it took you to graduate? You began your education in Fall semester and graduated in December.

Solution:

You began your education in Fall semester and graduated in December, so you received 5 payments in End of December (or first day of January) and 4 payments at the end of July (or first of August). To find the present worth (at time of graduation) you can find the future worth of each individual payment at time of graduation. You need to remember that periods are in months and number of periods (n) must be in months. Thus the payment you received for Fall semester before graduation has n=4. Since payments are equal we just need to add the (F/P, i, n) factors and multiply the results by $10,000. Also note that we need to calculate the rate of return per period.

i = 0.08/12 = 0.0066 = 0.66%

Sum = (F/P, 0.66%, 52) + (F/P, 0.66%, 48) + (F/P, 0.66%, 40) + (F/P, 0.66%, 36) + (F/P, 0.66%, 28) + (F/P, 0.66%, 20) + (F/P, 0.66%, 16) + (F/P, 0.66%, 8) + (F/P, 0.66%, 4)

Sum = 1.4078+1.3712+1.3009+1.2672+1.2022+1.1406+1.1109+1.0540+1.0266 = 10.8819
PW = $10,000 (10.8819) = $108,819.00

This value must be repaid over the same number of months (52) in equal payment.
$108,819 = A (P/A, 0.66%, 52)

$$\left(\frac{\mathrm{P}}{\mathrm{A}}, 0.66\%, 52\right) = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = \left[\frac{(1.0066)^{52} - 1}{0.0066(1.0066)^{52}}\right] = \left[\frac{0.4078}{0.0092}\right] = 43.8944$$

A = $108,819/43.8944 = $2479.10

Reality check: So, overall, you received 9 ($10,000) = $90,000 and you are paying back 52 (2479.10) = $128,913.58. Welcome to real life.