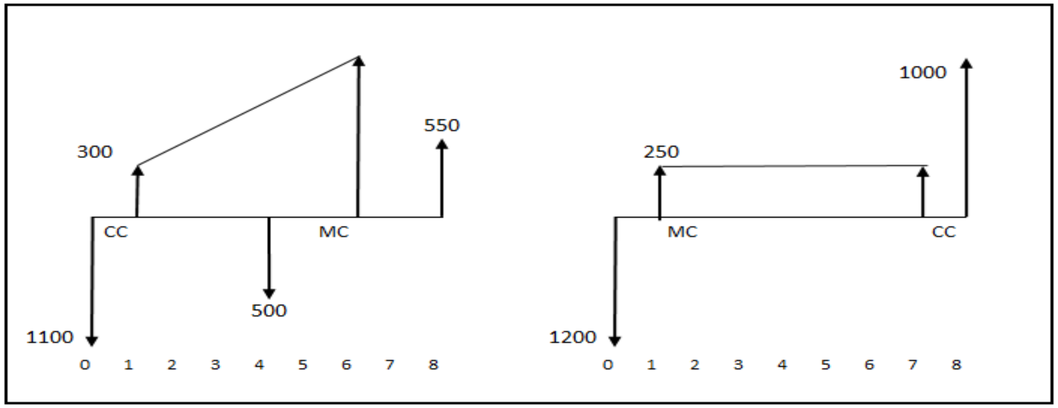
IEGR 350: Engineering Economy
Spring 2016
M. Salimian

Test 1
PROBLEM 1:
Given the two cash flows and for the interest rate of 8% compounded annually for the majority of project life times, calculate G that makes the two project present worth the same. Note the signs for monthly compounding (MC) and continuous compounding (CC).
(30 pts)

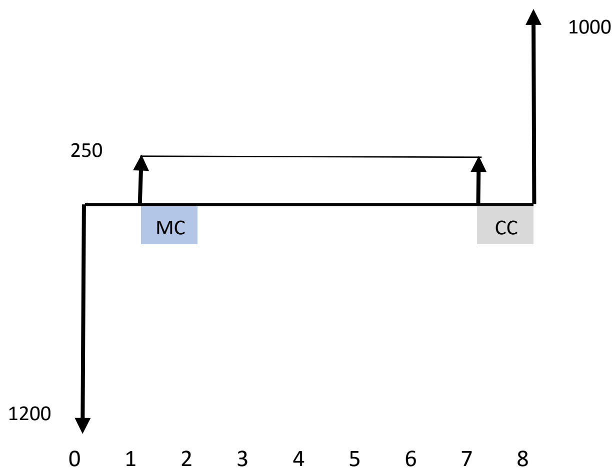


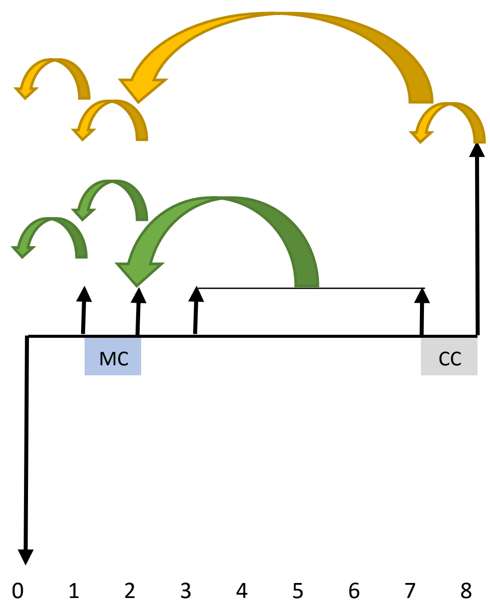
Analysis of the problem and selecting a solution approach
1. During the life time of both projects interest rate changes, we can not simply use the factors that are defined for homogenous interest rate to convert transactions which are in the future to to the present worth.
2. Project lives are the same (8 years).
3. All information is provided for both projects except the missing value of gradient in the first project which is unknown (G)
4. Knowing that, "the two project present worth the same", we will write the present worth of both projects, set them equal to each other and solve for missing G.

First, let's calculate the effective interest rates for different periods that have different compounding assumptions than yearly.

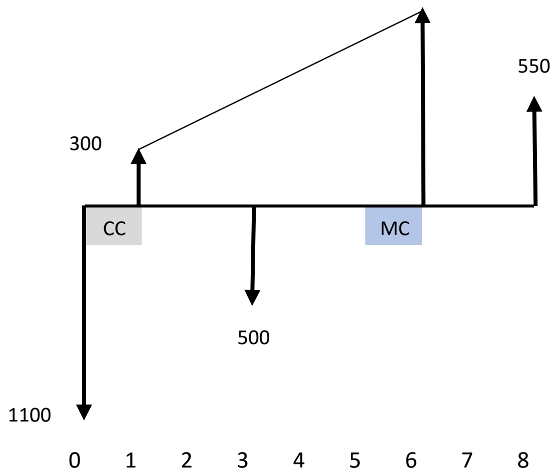
Effective Interest rate calculation:
Nominal: 8%, (MC) monthly compounding
$r = (1 + i/n)^{n} - 1 = (1+0.08/12)^{12} - 1 =$
$1.083 - 1 = 0.083$ or 8.3%

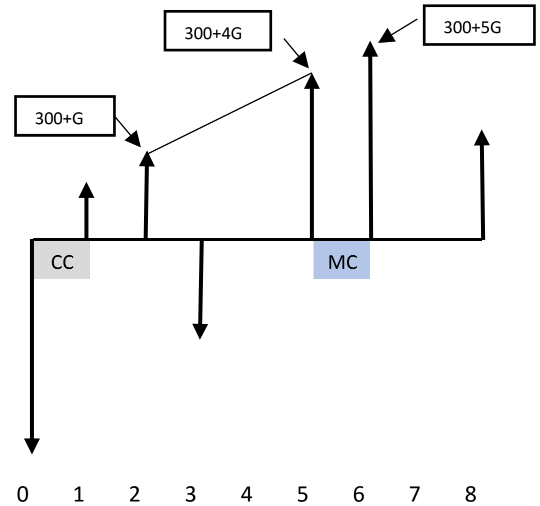
Effective Interest rate calculation:
Nominal: 8%, (CC) continuous compounding
$r = e^{i} - 1 = (2.718)^{0.08} - 1 =$
$1.0833 - 1 = 0.0833$ or 8.33%

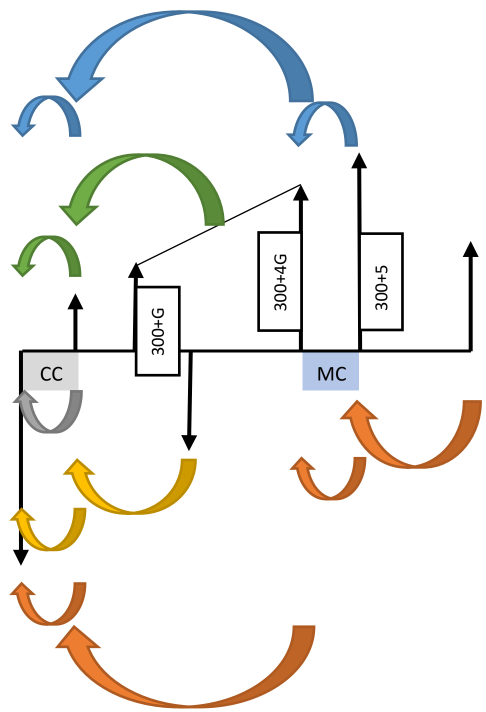




Note that this can be written as one long statement but I have done it for the individual transactions for additional clarity.
PW2 = - 1200 + 250 (P/F, 8%, 1) + 250 (P/F, 8.3%, 1)(P/F, 8%, 1) +
250 (P/A, 8%, 5)(P/F, 8.3%, 1)(P/F, 8%, 1) +
1000 (P/F, 8.33%, 1)(P/F, 8%, 5)(P/F, 8.3%, 1)(P/F, 8%, 1)
PW2 = - 1200 + 250 (0.9259) + 250 (0.9234)(0.9259) +
250 (3.9582)(0.9234)(0.9259) +
1000 (0.9231)(0.6703)(0.9234)(0.9259) = $620.28


550
300
CC
MC
500
1100
0
1
2
3
4
5
6
7
8


300+4G
300+5G
300+G
CC
MC
0
1
2
3
4
5
6
7
8


300+4G
300+5
300+G
CC
MC

Using the diagram above:

PW1 = - 1100 + 300 (P/F, 8.33%, 1) + (300+G)(P/A, 8%, 4)(P/F, 8.33%, 1) +
G (P/G, 8%, 4)(P/F, 8.33%, 1) - 500 (P/F, 8%, 2)(P/F, 8.33%, 1) +
(300+5G)(P/A, 8.3%, 1)(P/F, 8%, 4) (P/F, 8.33%, 1) +
550 (P/F, 8%, 2)(P/F, 8.3%, 1) (P/F, 8%, 4)(P/F, 8.33%, 1)

PW1 = - 1100 + 300 (0.9231) + (300+G)(3.3121)(0.9231) +
G (4.6501)(0.9231) - 500 (0.8573)(0.9231) +
(300+5G)(0.9234)(0.7350) (0.9231) +
550 (0.8573)(0.9234) (0.7350)(0.9231) = 181.82 + 10.4824 G

PW1 = PW2 = 181.82 + 10.4824 G = 620.28

G = (620.28 - 181.82) / 10.4824 = 41.83

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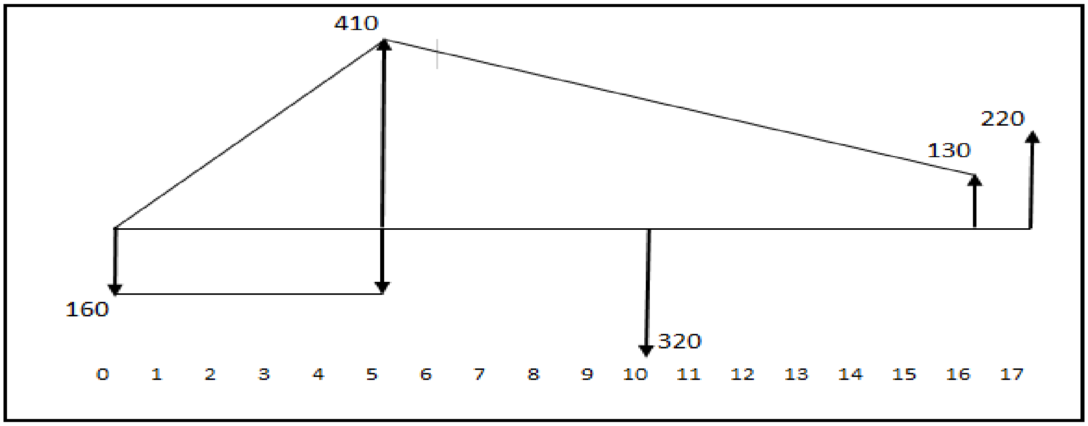
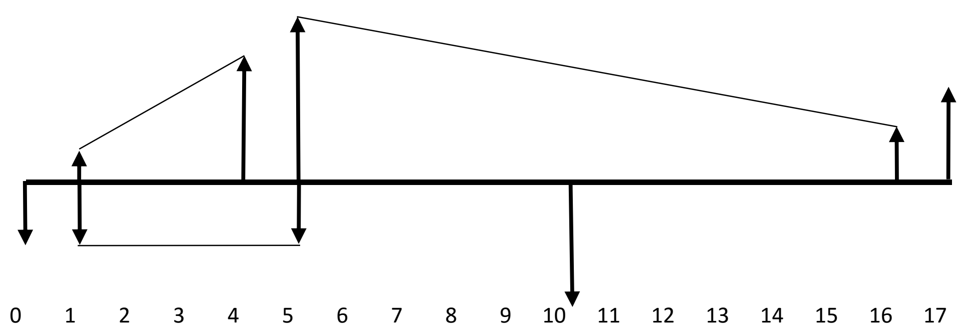
Test 1
PROBLEM 2:
Find the rate of return for the following cash flow with annual compounding. (40 pts)

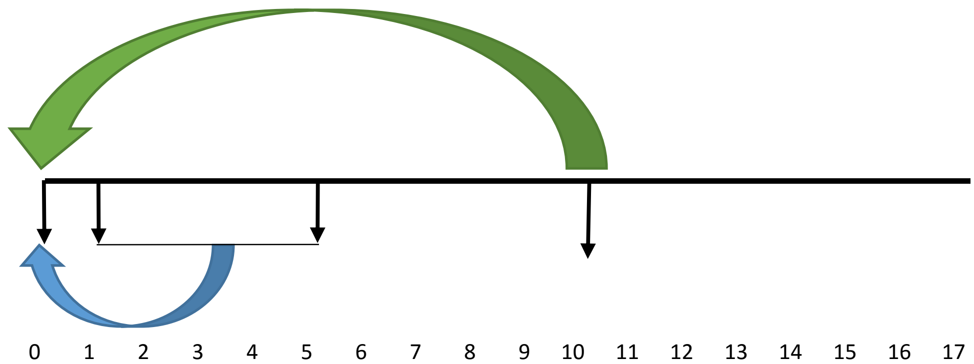


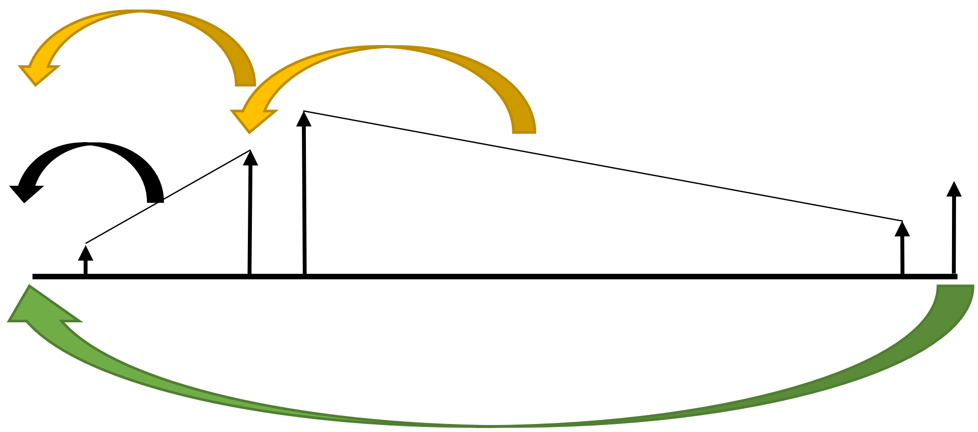
Analysis of the problem and selecting a solution approach

1. There is a shared transaction at year 5 between the two arithmetic gradient series that we need to separate the two series and assign that payment to only one of them.
2. We also need to separate the payment in year 0 from the series for present worth calculations.
3. Then, find the present worth of the series. Since, we do not know the interest rate, the present worth will be written in full involving conversion factors P/F, P/A, and P/G.
4. using trial and error, we find the interest rate at which present worth become 0.



Calculating the gradients and initial values of gradient series:
G1 = (410-0)/5 = 82 (which is also the A for the first gradient series.
G2 = (410-130)/11 = 280/11 = 25.45 (A for this series is 410).





P1 = - 160 - 160 (P/A, i, 5) - 320 (P/F, i, 10)
P2 = 82 (P/A, i, 4) + 82 (P/G, i, 4) + [410 (P/A, i, 12) - 25.45 (P/G, i, 12)] (P/F, i, 4) + 220 (P/F, i, 17)
PW = P1 + P2

Now we begin searching for the interest rate that makes PW =0 using trial and error. To get an idea of what interest rate value to begin with, we can begin with zero interst rate.
PW (i=0) = - 960 - 320 + 82 + 164 + 246 + 328 + 410 + 384.55 + 359.09 + 333.64 + 308.18 + 282.73
+ 257.27 + 231.82 + 206.36 + 180.91 + 155.45 + 130 + 220 = 3000
which is quite large in relation to the size of payments, so we have to begin with larger interest rates.
Using tables: PW (i=12%) = 974.02, and PW (i=15%) = + 752.03
Note that during the test there was a limited access to the tables thus, at this point, we could either use extrapolation or do the calculation using the formulas. Due to the large size of PW extrapolation is not recommended. But here is how it is done:
From 12% to 15% (3% difference) we had 974.02 - 752.03 = 221.99
So, for each 1%, there is 221.99/3 = 74 change in present worth. To get to zero, we need to decrease another 752.03, thus: 752.03/74 = 10.16 Therfore, the rate of return is 15 + 10.16 = 25.16%
This value is far from the real rate of return as is shown below.
Better solution is obtained using formulas for much larger interest rate, say 50%
PW (i=50%) = - 36.25 and then interpolating between 15% and 50% .
which would yield : [(752.03+36.25)/(50-15)] = 22.52 and 752.03/22.52 = 33.39 or i* = 15+33.49 = 48.92%
Similar to the extrapolation case, the number is not accurate either due to large differences, Interpolation leads better results over short ranges. True rate of return is 44.92%

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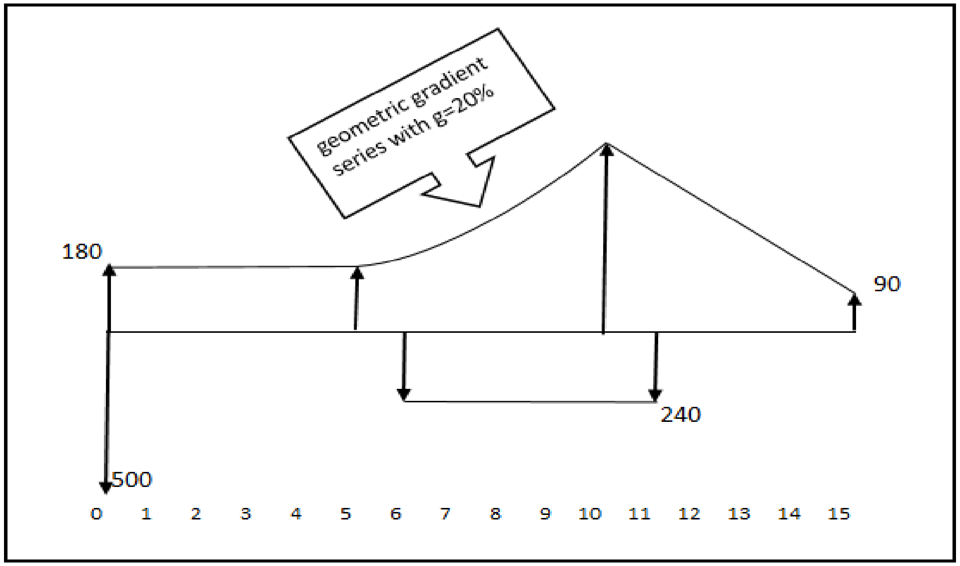
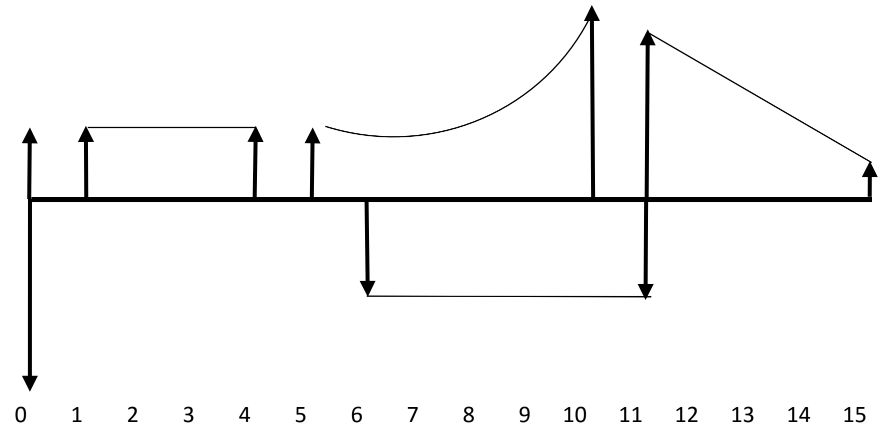
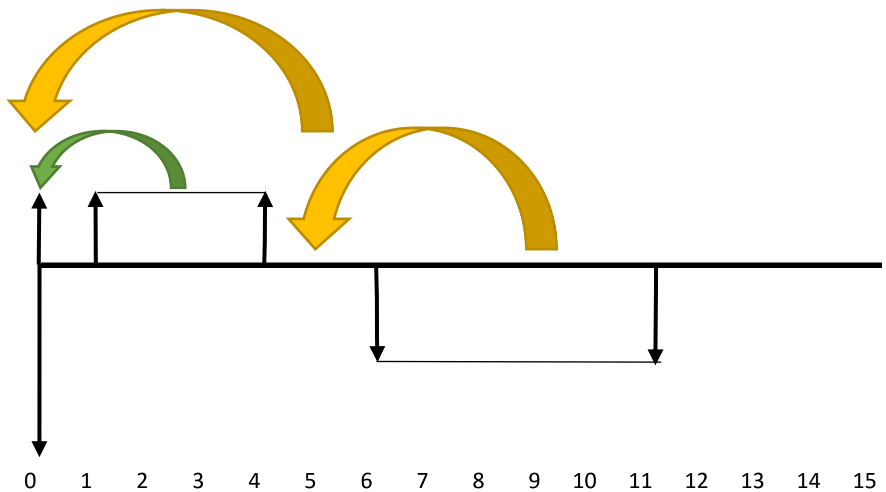
Test 1
PROBLEM 3:
Find the equivalent worth of the following cash flow at year 8. Interest rate is 15%, compounded annually. (30 pts)



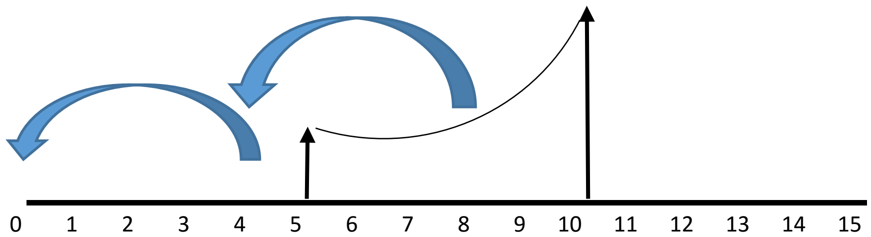
Analysis of the problem and selecting a solution approach
1. There are shared transaction at years 5 and 10 that we need to separate the series and assign those payments to only one of them.
2. We also need to separate the payment in year 0 from the series for present worth calculations.
3. Then, find the present worth of the series and find the future worth of that value at year 8.
4. Payment at year 10 is 180 $(1.20)^5$ = 447.90
5. Arithmetic gradient value is (447.90 - 90) / 5 = 71.58
6. Payment at year 11 is 447.90 - 71.58 = 376.32



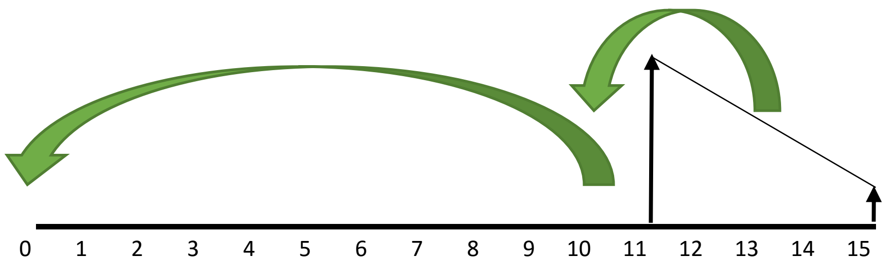


PW1 = - 500 + 180 + 180 (P/A, 15%, 4) - 240 (P/A, 15%, 6) (P/F, 15%, 5)

PW1 = - 500 + 180 + 180 (2.8550) - 240 (3.7845) (0.4972) = - $257.70



PW2@yr4 = 180 (P/g, 20%, 15%, 6)
= 180 [1 – ((1 + 0.2)/(1 + 0.15))6] / (0.15 – 0.2)
= 180 [1 - (1.2/1.15)6] / (- 0.05) = 1047.32
PW2@yr0 =1074.32 (P/F, 15%, 4) = 1074.32 (0.5718) = $598.86



PW3@yr10 = 376.32 (P/A, 15%, 5) - 71.58 (P/G, 15%, 5)
= 376.32 (3.3522) - 71.58 (5.7751) = 848.12
PW3@yr0 =848.12 (P/F, 15%, 10) = 848.12 (0.2472) = $209.65

The equivalent value of the cash flow at year 0 is:
PW@yr0 = PW1 + PW2 + PW3 = - 257.70 + 598.86 + 209.65 = $550.81
The equivalent value of the cash flow at year 8 is:
FW@yr8 = 550.81 (F/P, 15%, 8) = 550.81 (3.0590) = $1684.93