

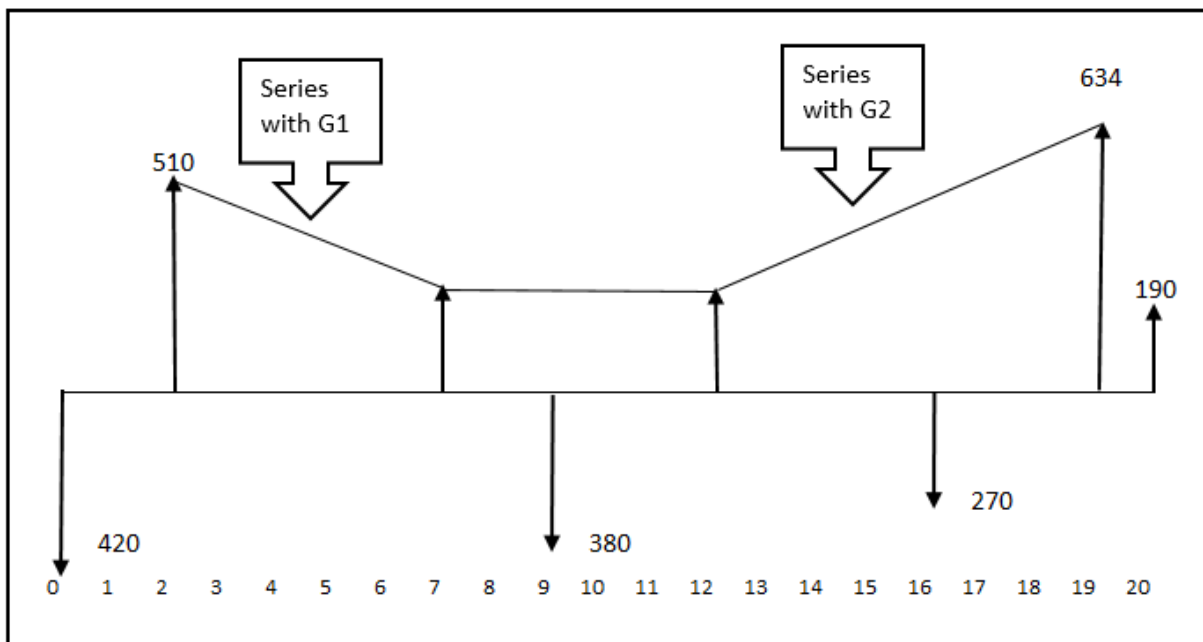
Quiz 2

100 Points (Time: 30:00 Minutes)

Explain and show your work. No use of computer or cell phone allowed. Use of printed formula sheet and table for 12% is allowed. Non-digital textbook for use of 12% table (not the formula) is allowed.

PROBLEM:

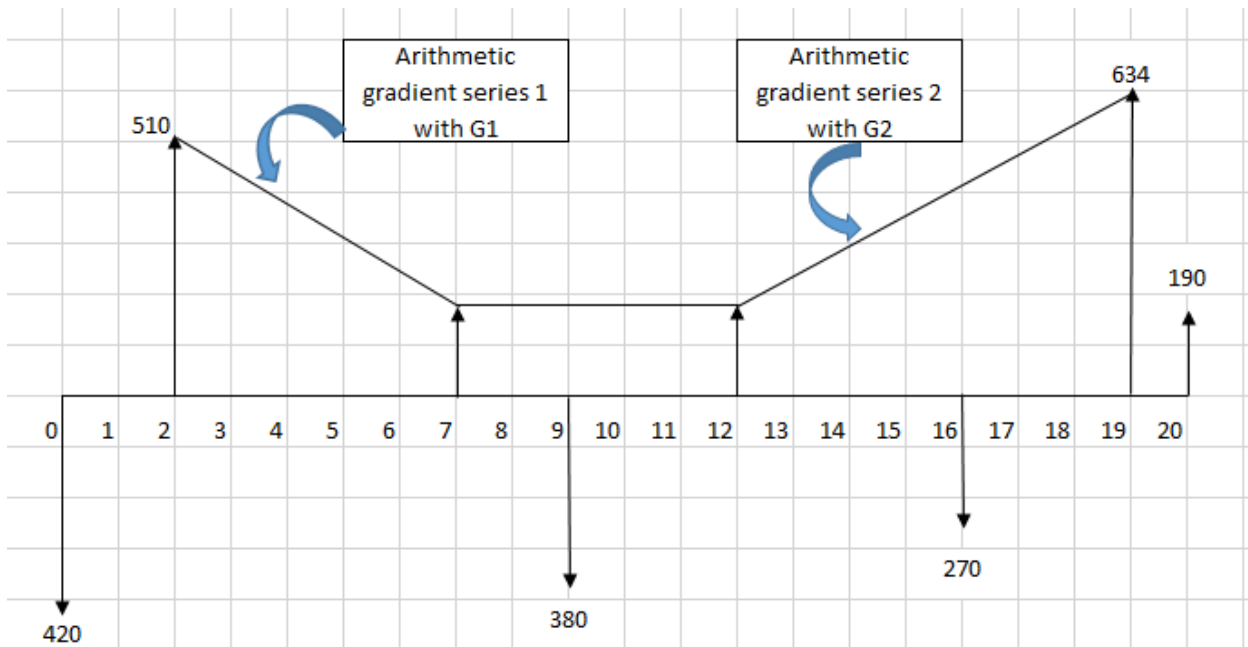
1. Given the cash flow and for the interest rate of 12% compounded annually, calculate $G1$ and $G2$ if the present worth of the project is \$1880.93 and the combined values of the gradients is \$100. (80 pts)
2. Engineering Economy tables usually provide only the $(P/G, i, n)$ factor values. Develop a formula for $(F/G, i, n)$.



SOLUTION KEY:

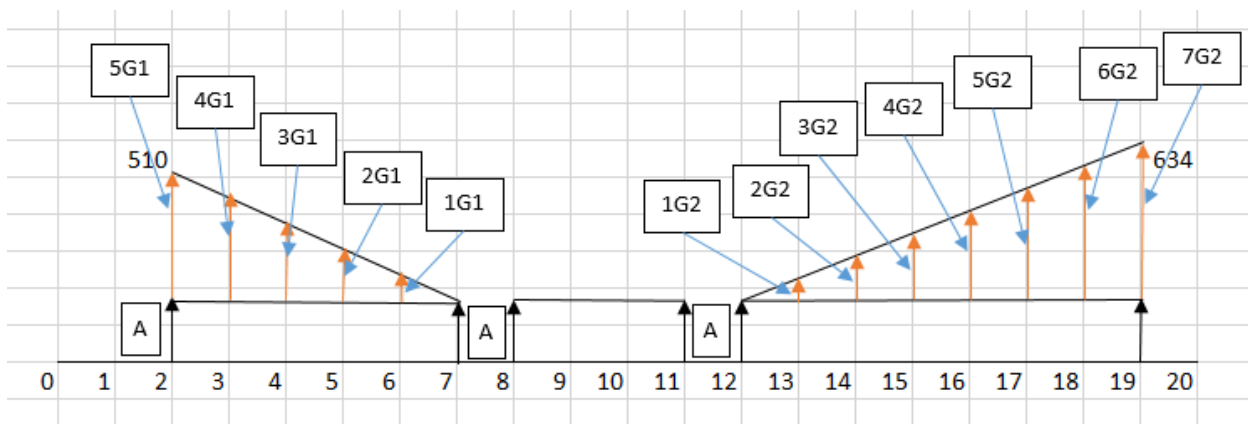
When analyzing the cash flow, we notice that payments at year 7 and 12 are common between two series. And we must make a decision of assigning them to only one of the series. In here, I have assigned them to the gradient series.

Your Name:



The present worth of the cash flow is given as 1880.93, so we can find the present worth of the cash flow and set it equal to the given value of 1880.93. However, the present worth of the first gradient series will involve an unknown value of G_1 and the second series will involve a not only an unknown value of G_2 but also an unknown value of A (initial payment of second gradient series which is the same for the annuities too).

Can we find A based on G_1 or G_2 ? Let's look at the first series.



Initial payment of the first gradient series can be written as $510 = 5G_1 + A$

Solving for A, we have:

$$A = 510 - 5 G_1$$

Now the present worth of the complete cash flow can be written as:

$$P_0 = - \mathbf{420}$$

$$P_1 = [510 (P/A, 12\%, 6) - G_1 (P/G, 12\%, 6)] (P/F, 12\%, 1)$$

$$[510 (4.1114) - G_1 (8.9302)] (0.8929) = \mathbf{1872.24 - 7.9738 G_1}$$

$$P_2 = (510 - 5 G_1) (P/A, 12\%, 4) (P/F, 12\%, 7)$$

$$(510 - 5 G_1) (3.0373) (0.4523) = \mathbf{700.62 - 6.8689 G_1}$$

$$P_3 = - 380 (P/F, 12\%, 9) = - 380 (0.3606) = - \mathbf{137.03}$$

$$P_4 = [(510 - 5 G_1) (P/A, 12\%, 8) + G_2 (P/G, 12\%, 8)] (P/F, 12\%, 11) =$$

$$[(510 - 5 G_1) (4.9676) + G_2 (14.4714)] (0.2875) =$$

$$[2533.47 - 24.838 G_1 + 14.4714 G_2] (0.2875) =$$

$$\mathbf{728.37 - 7.1409 G_1 + 4.1605 G_2}$$

$$P_5 = - 270 (P/F, 12\%, 16) = - 270 (0.1631) = - \mathbf{44.04}$$

$$P_6 = 190 (P/F, 12\%, 20) = 190 (0.1037) = \mathbf{19.70}$$

$$PW = - 420 + 1872.24 - 7.9738 G_1 + 700.62 - 6.8689 G_1 - 137.03$$

$$+ 728.37 - 7.1409 G_1 + 4.1605 G_2 - 44.04 + 19.70 =$$

$$2719.86 - 21.9831 G_1 + 4.1605 G_2$$

Setting it equal to 1880.93 we will have an equation involving two unknowns G_1 and G_2 .

$$2719.86 - 21.9831 G_1 + 4.1605 G_2 = 1880.93$$

$$- 21.9831 G_1 + 4.1605 G_2 = 1880.93 - 2719.86 = - 838.93$$

Additionally, we are provided with the information that $G_1 + G_2 = 100$.

Solving the linear system of equations, we will have,

$$- 21.9831 G_1 + 4.1605 G_2 = - 838.93$$

$$G_1 + G_2 = 100$$

Find G1 from second equation and substitute in the first equation.

$$G1 = 100 - G2$$

$$- 21.9831 G1 + 4.1605 (100 - G1) = - 838.93$$

$$- 21.9831 G1 + 416.05 - 4.1605 G1 = - 838.93$$

$$- 26.1436 G1 = - 1254.98$$

$$G1 = 48$$

$$G2 = 100 - 48 = 52$$

Part 2: Developing a formula for (F/G, i, n)

The easiest and most convenient approach is to find the present worth using the (P/G, i, n) and then find the future worth of that value using (F/P, i, n) factor. Formula for (P/G, i, n) is given as:

$$\left(\frac{P}{G}, i, n\right) = \frac{(1+i)^n - in - 1}{i^2(1+i)^n}$$

To find its future value at year n, we just need to multiply it by $(1+i)^n$

Which will be crossed out by the same value in the denominator. So,

$$\left(\frac{F}{G}, i, n\right) = \frac{(1+i)^n - in - 1}{i^2}$$