IEGR 350: Engineering Economy
Fall 2015
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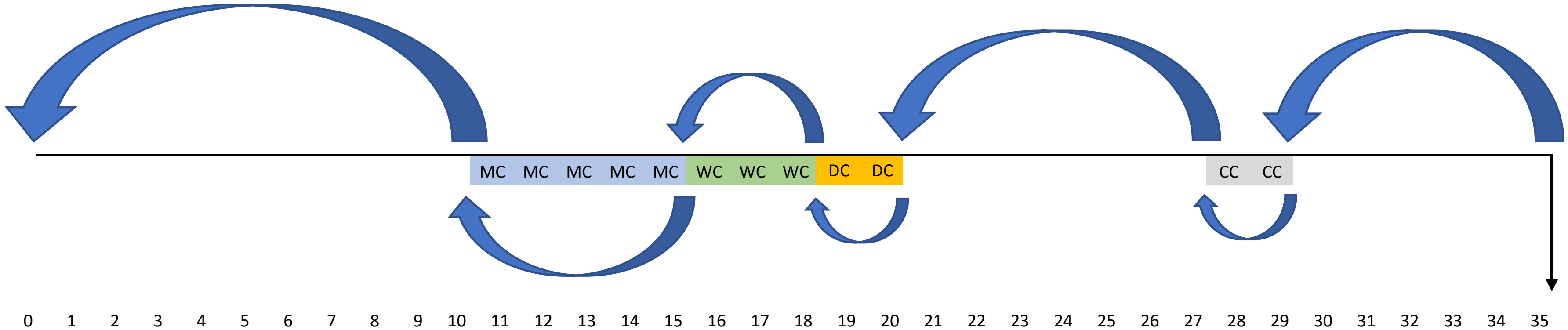
Assignment 5
Use formulas only, no tables.
PROBLEM:
For a 35-year major construction project the following information is provided (values in 1000 dollars):
- $4,200 now and yearly initial investment through year 5 decreasing by $200
- $3,900 investment in years 10, 20, and 30
- $4,500 investment to tear down and clean up at the end of project life
- $3,700 revenue from year 5 to 10, decreasing by $300 each year
- $300 monthly revenue between years 11 through 15
- $50 weekly revenue between years 16 through 18
- $10 daily revenue for years 19 and 20
- $3,000 annual revenue in year 21 increasing by 15% yearly through year 28, then decreasing annually by a fixed amount to $1,000 on the last year of project life
- $350 revenue from the sale of scraps from project at the end of project life

Annual compounding interest rate of 9.5% is in effect for the majority of project life except for the following years: (WC)- years 16 through 18, (MC)- years 11 through 15, (DC) years 19 and 20, and (CC) years 28 and 29
(1) Find out the future worth of the cash flow at the end of the project (50 pts)
(2) If the project only used annual compounding throughout the life of the project, what would the present worth and the rate of

Since during the life time of the project interest rate changes, we can not simply use the factors that are defined for homogenous interest rate to convert transactions which are in the future to to the present worth. As an example, consider the $4,500 investment to tear down and clean up at the end of project life. If interest rate stayed the same throughout the life of the project, 4500(P/F, i, 35) would result in the present worth of that transaction. However, due to change in the interest rate, the present worth involved several conversion as demonstrated in the illustration below.



First, let's calculate the effective interest rates for different periods that have different compounding assumptions than yearly.

Effective Interest rate calculation:
Nominal: 9.5%, (MC) monthly compounding
$r = (1 + i/n)^{n} - 1 = (1+0.095/12)^{12} - 1 =$
$1.09925 - 1 = 0.09925$ or 9.92%

Effective Interest rate calculation:
Nominal: 9.5%, (WC) weekly compounding
$r = (1 + i/n)^{n} - 1 = (1+0.095/52)^{52} - 1 =$
$1.09956 - 1 = 0.09956$ or 9.95%

Effective Interest rate calculation:
Nominal: 9.5%, (DC) daily compounding
$r = (1 + i/n)^{n} - 1 = (1+0.095/365)^{365} - 1 =$
$1.09964 - 1 = 0.09964$ or 9.96%

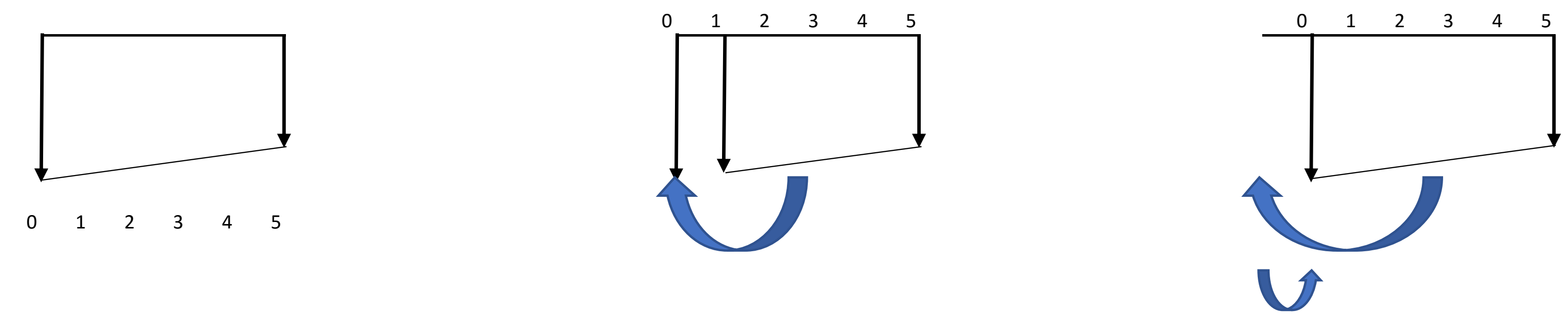
Effective Interest rate calculation:
Nominal: 9.5%, (CC) continuous compounding
$r = e^{i} - 1 = (2.718)^{0.095} - 1 = 1.0966 - 1 = 0.0966$ or 9.96%

Now, we can write the present worth of the $4500 at year 35 as:
4500 (P/F, 9.5%, 6)(P/F, 9.96%, 2)(P/F, 9.5%, 7)(P/F, 9.96%, 2)(P/F, 9.95%, 3)(P/F, 9.92%, 5)(P/F, 9.5%, 10)

Below is the cash flow representation of "$4,200 now and yearly initial investment through year 5 decreasing by $200". There are two ways we can handle this.

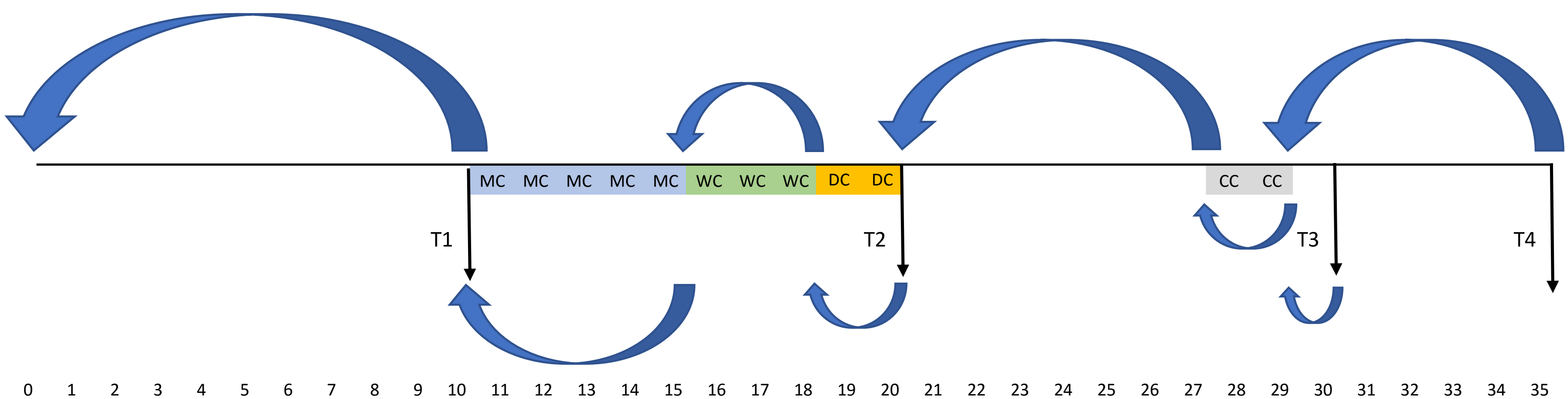
Separate the first transaction from the rest, then find the present worth of the gradient series and add it to the first one.

Find the present worth of the series as it is and the find the future worth of that amount after one year. This method may not be suitable here because we have no information about compounding prior to year 0.



Using the first method, we can write the present worth as:
- 4200 - 4000 (P/A, 9.5%, 5) + 200 (P/G, 9.5%, 5)

Below is the cash flow representation of several individual transactions at different years:
$3,900 investment in years 10, 20, and 30 (T1, T2, T3)
$4,500 investment to tear down and clean up at the end of project life
$350 revenue from the sale of scraps from project at the end of project life
Note: year 35 values can be converted into one value (-4500+350=-4150) (T4)



Note that this can be written as one long statement but I have done it for the individual transactions for additional clarity. **2**

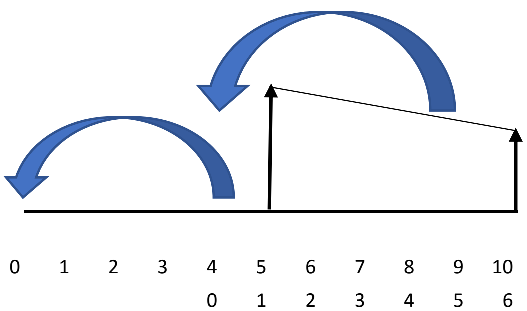
T1 (P/F, 9.5%, 10) +

T2 (P/F, 9.66%, 2)(P/F, 9.95%, 3)(P/F, 9.92%, 5)(P/F, 9.5%, 10) +

T3 (P/F, 9.5%, 1)(P/F, 9.96%, 2)(P/F, 9.5%, 7)(P/F, 9.66%, 2)(P/F, 9.95%, 3)(P/F, 9.92%, 5)(P/F, 9.5%, 10) +

T4 (P/F, 9.5%, 6)(P/F, 9.96%, 2)(P/F, 9.5%, 7)(P/F, 9.66%, 2)(P/F, 9.95%, 3)(P/F, 9.92%, 5)(P/F, 9.5%, 10)

Below is the cash flow representation of " $3,700 revenue from year 5 to 10, decreasing by $300 each year".



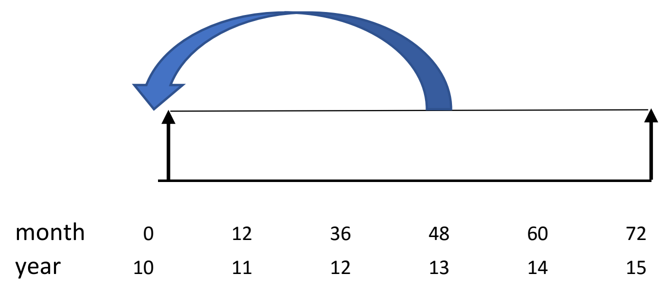
Present worth of the arithmetic gradient series is: **3**

{ 3700 (P/A, 9.5%, 6) - 300 (P/G, 9.5%, 6) } (P/F, 9.5%, 4)

Up to this point we were able to use annual values using either nominal or effective annual interest rate because the transactions occurred at the end of each year and compoundings were annually.

For the " $300 monthly revenue between years 11 through 15" we are dealing with two new scenarios: monthly transactions and monthly compounding. Since, throughout years 10 to 15 all compounding is the same, we can consider each period to be a month instead of a year.

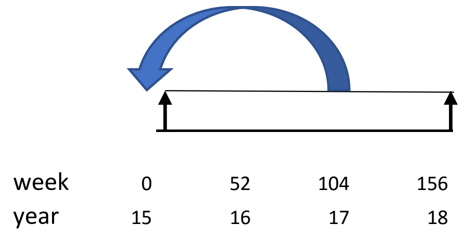
It is important to recognize three points: (1)- we have 72 payments from year 11 to year 15 and the first payment is at the end of the first month after year 10, (2)- interest rate per period (month) is 0.095/12=0.0079, and (3)- P/A factor converts the payment to a value at the end of year 10.



Present worth of the monthly payments is: **4**

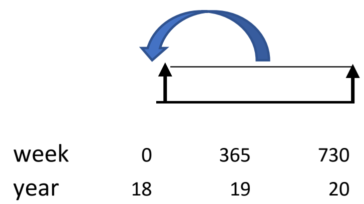
300 (P/A, 0.79%, 72)(P/F, 9.5%, 10)

The next case is similar to the above case except that payments are weekly and thoughtout the periods (weeks) compounding remains fixed as weekly.
"$50 weekly revenue between years 16 through 18"
It is important to recognize four points: (1)- we have 156 payments from year 16 to year 18 and the first payment is at the end of the first week after year 15, (2)- interest rate per period (week) is 0.095/52=0.0018, and (3)- P/A factor converts the payment to a value at the end of year 15, and (4)- years 11-15 have MC and thus their effective interest rate must be considered.



Present worth of the weekly payments is:
50 (P/A, 0.18%, 156)(P/F, 9.92%, 5)(P/F, 9.5%, 10) 5

The next case is similar to the above cases except that payments are daily and thoughtout the periods (days) compounding remains fixed as daily.
"$10 daily revenue between years 19 through 20"
It is important to recognize four points: (1)- we have 730 payments from year 19 to year 20 and the first payment is at the end of the first day after year 18, (2)- interest rate per period (day) is 0.095/365=0.00026, and (3)- P/A factor converts the payment to a value at the end of year 18, and (4)- years 11-15 have MC and years 16-18 have WC thus their effective interest rate must be considered.



Present worth of the weekly payments is:
10 (P/A, 0.026%, 730)(P/F, 9.95, 3)(P/F, 9.92%, 5)(P/F, 9.5%, 10) 6

The next case combines two challenges: one is a joint payment at year 28 that we must make a decision to as to which series to assign it to and second, the nonhomogenous compounding for years 28 and 29.
We should consider payments at years 28 and 29 as individual payments and end the geometric gradient series at year 27. Furthermore, the arithmetic series will have years 30-35 with the same interest rate and compounding structure. ,Don't forget that from the total result will be a value at the end of year 20 that to find its present worth you must consider effective rates in previous years too.

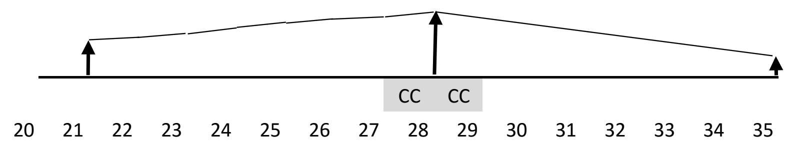
Payment at year 21 = 3000

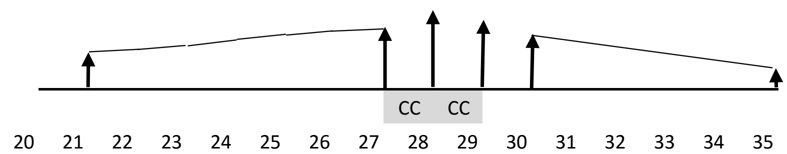
Payment at year 27 = 3000 $(1.15)^6$ = 6939.18

Payment at year 28 = 3000 $(1.15)^7$ = 7980.06

Payment at year 29 = 7980.06-1000 = 6980.06

Payment at year 30 = 6980.06-1000 = 5980.06





Geometric gradient series: 7

P1 at year 20 = 3000 (P/A1, 15%, 9.5%, 7) =

$3000 \{[1- (1.15/1.095)^7]/(0.095-0.15)\}$ =

Two individual payments:

P2 at year 20 = 7980.06 (P/F, 9.96%, 1)(P/F, 9.5, 7) =

P3 at year 20 = 6980.06 (P/F, 9.96%, 2)(P/F, 9.5, 7) =

Arithmetic gradient series:

P4 at year 29 = 5980.06 (P/A, 9.5%, 6) + 1000 (P/G, 9.5%, 6)

P4 at year 20 = (P4 at year 29)(P/F, 9.96%, 2)(P/F, 9.5, 7) =

Total at year 20

W = P1 + P2 + P3 + P4

Present worth at year 0:

W(P/F, 9.96%, 2)(P/F, 9.95%, 3)(P/F, 9.92%, 5)(P/F, 9.5%, 10)

Present worth of the project is the addition of result above

PW = 1 + 2 3 4 5 6 7

Calculating future worth at year 35:

PW (F/P, 9.5%, 10)(F/P, 9.92%, 5)(F/P, 9.95%, 3)(F/P, 9.96%, 2)(F/P, 9.5%, 7)(F/P, 9.96%, 2)(F/P, 9.5%, 6)

In question number 2 problem becomes significantly easier considering the fact no more MC, WC, and DC needs to be applied to monthly, weekly and daily payments. Instead one interest rate of 9.5% is in effect for the entire project life. But, what about the payments that are received daily, weekly, or monthly? They will treated like any other payment throughout that year which means they will be added together and one value will be represented at the end of the year. For example, $300 monthly revenue between years 11 through 15 will be an annuity of (300)(12) for each year or, $50 weekly revenue between years 16 through 18 is an annuity of (50)(52) for that period. Solution is left as practice for you.