

EXAMPLES - Uniform Series That Are Shifted

EXAMPLE-1

the offshore design group at Bechtel just purchased upgraded CAD software for \$5000 now and annual payments of \$500 per year for 6 years starting 3 years from now for annual upgrades. What is the present worth in year 0 of the payments if the interest rate is 8% per year?

Solution

The cash flow diagram is shown in Figure 3-4. The symbol P_A is used throughout this chapter to represent the present worth of a uniform annual series A , and P'_A represents the present worth at a time other than period 0. Similarly, P_T represents the total present worth at time 0. The correct placement of P'_A and the diagram renumbering to obtain n are also indicated. Note that P'_A is located in actual year 2, not year 3. Also, $n=6$, not 8, for the P/A factor. First, find the value of P'_A of the shifted series.

$$P'_A = \$500(P/A, 8\%, 6)$$

Since P'_A is located in year 2, now find P_A in year 0.

$$P_A = P'_A(P/F, 8\%, 2)$$

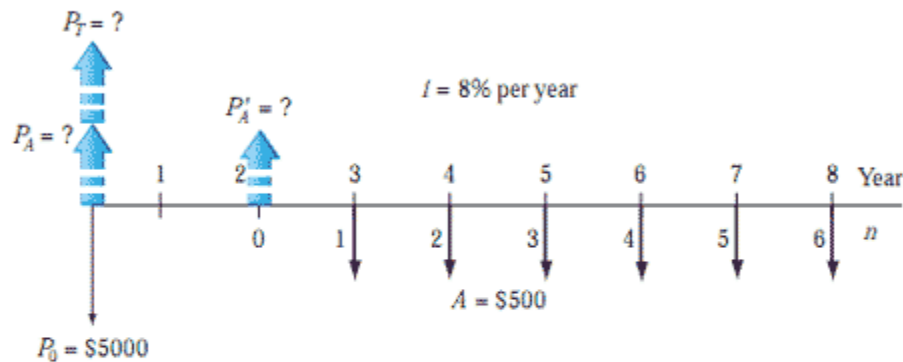


Figure 3-4

Cash flow diagram with placement of P values, Example 3.1.

The total present worth is determined by adding P_A and the initial payment P_0 in year 0.

$$\begin{aligned} P_T &= P_0 + P_A \\ &= 5000 + 500(P/A, 8\%, 6)(P/F, 8\%, 2) \\ &= 5000 + 500(4.6229)(0.8573) \\ &= \$6981.60 \end{aligned}$$

EXAMPLE-2

Recalibration of sensitive measuring devices costs \$8000 per year. If the machine will be recalibrated for each of 6 years starting 3 years after purchase, calculate the 8-year equivalent uniform series at 16% per year. Show hand and spreadsheet solutions.

Solution by Hand

Figure 3–5 a and b shows the original cash flows and the desired equivalent diagram. To convert the \$8000 shifted series to an equivalent uniform series over all periods, first convert the uniform series into a present worth or future worth amount. Then either the A/P factor or the A/F factor can be used. Both methods are illustrated here.

Present worth method. (Refer to Figure 3–5a.) Calculate P'A for the shifted series in year 2, followed by P_T in year 0. There are 6 years in the A series.

$$\begin{aligned} P'_A &= 8000(P/A, 16\%, 6) \\ P_T &= P'_A(P/F, 16\%, 2) = 8000(P/A, 16\%, 6)(P/F, 16\%, 2) \\ &= 8000(3.6847)(0.7432) = \$21,907.75 \end{aligned}$$

The equivalent series A' for 8 years can now be determined via the A/P factor.

$$A' = P_T(A/P, 16\%, 8) = \$5043.60$$

Future worth method. (Refer to Figure 3–5a.) First calculate the future worth F in year 8.

$$F = 8000(F/A, 16\%, 6) = \$71,820$$

The A/F factor is now used to obtain A' over all 8 years.

$$A' = F(A/F, 16\%, 8) = \$5043.20$$

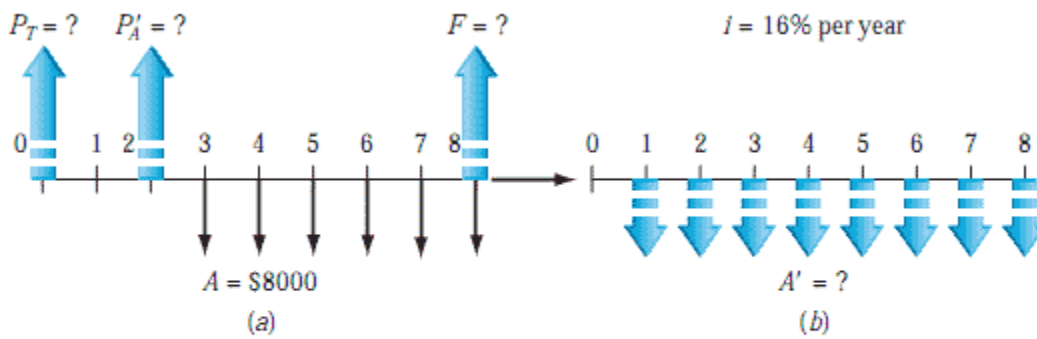


Figure 3-5

(a) Original and (b) equivalent cash flow diagrams; and (c) spreadsheet functions to determine P and A , Example 3.2.

Source:

<http://engineeringandeconomicanalysis.blogspot.com/2014/05/examples-uniform-series-that-are-shifted.html>