Midterm Exam – Time: 90 Minutes

Problem 1 (15 points)

You roll a die 5 times. What is the probability that at least one value is observed more than once?

Problem 2 (10 points)

What should the values of c and d be for the following function to be a legitimate probability distribution function if we know that the E(X) = 5/2?

$$P_X(k) = \begin{cases} c & \text{for } k = 1 \\ \frac{1}{8} & \text{for } k = 2 \\ \frac{1}{3} & \text{for } k = 3 \\ d & \text{for } k = 4 \\ 0 & \text{otherwise} \end{cases}$$

Problem 3 (20 Points)

Let X be a discrete random variable with the following PMF

$$P_X(k) = \begin{cases} 0.5 & \text{for } k = 1 \\ 0.3 & \text{for } k = 2 \\ 0.2 & \text{for } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find EX.

(b) Find $\text{Var}(X)$, and $SD(X)$.

(c) If $Y = \frac{2}{X}$, find EY.

Problem 4 (20 Points)

Let $X \sim N(3, 9)$.

    a. Find $P(X > 0)$.
    b. Find $P(-3 < X < 8)$.
    c. Find $P(X > 5 | X > 3)$.

Problem 5 (15 Points)

$X \sim$ Poi (4.8). Find $P (3 \leq X \leq 7)$.

Problem 6 (20points)

Let $X \sim Exponential(4)$ and $Y = 4+3X$.

   a. Find $P(X > 2)$.
   b. Find EY and $Var(Y)$.
   c. Find $P(X > 2|Y < 11)$.