Assignment 2: Basic Probability and distributions

Resource:

H. Pishro-Nik, "Introduction to probability, statistics, and random processes", available at https://www.probabilitycourse.com, Kappa Research LLC, 2014.

Question 1: Discrete Probability (15 points)

Consider a random experiment with a sample space, $S = \{1, 2, 3, \dots\}$

Suppose that we know:

$$P(k) = P(\{k\}) = \frac{c}{3^k}$$
 for $k = 1, 2, \cdots$

where c is a constant number.

- (a) Find c.
- (b) Find P ({2, 4, 6}).
- (c) Find P ($\{3, 4, 5, \cdots\}$).

Question 2: Continuous Probability (15 points)

Let T be the time needed to complete a job at a certain factory. By using the historical data, we know that

$$P(T \le t) = \begin{cases} \frac{1}{16}t^2 & \text{for } 0 \le t \le 4\\ 1 & \text{for } t \ge 4 \end{cases}$$

(a) Find the probability that the job is completed in less than one hour, i.e., find $P(T \le 1)$.

(b) Find the probability that the job needs more than 2 hours.

(c) Find the probability that $1 \le T \le 3$.

Question 3: Conditional Probability (5 points)

Suppose that of all the customers at a coffee shop:

-70% purchase a cup of coffee.

-40% purchase a piece of cake.

-20% purchase both a cup of coffee and a piece of cake.

Given that a randomly chosen customer has purchased a piece of cake, what is the probability that he/she has also purchased a cup of coffee?

Question 4: Uniform Distribution (15 points)

A real number X is selected uniformly at random in the continuous interval [0, 10].

(a) Find P ($2 \le X \le 5$).

(b) Find P ($X \le 2 | X \le 5$).

(c) Find P $(3 \le X \le 8 | X \ge 4)$.

Question 5: Use EXCEL for counting (10 points)

Suppose that x_i's must be non-negative integers, i.e.,

 $x_i \in \{0, 1, 2, \cdot \cdot \cdot\}$ for i = 1, 2, 3.

(a) How many distinct solutions does the equation x1 + x2 + x3 = 100 have such that at least one of the x_i's is larger than 40?

(b) How many distinct solutions does the equation x1 + x2 + x3 = 100 have such that at least two of the x_i's are larger than 30?

Question 6: Discrete Probability (15 points)

Let X and Y be two independent discrete random variables with the following PDFs.

- a. Find P (X \leq 2 and Y \leq 2).
- b. Find P (X>2 or Y>2).
- c. Find P(X < Y).

$P_X(k) = \langle$	$ \left(\begin{array}{c} \frac{1}{4} \\ \frac{1}{8} \\ \frac{1}{2} \\ 0 \end{array}\right) $	for $k = 1$ for $k = 2$ for $k = 3$ for $k = 4$ otherwise
$P_Y(k) = \langle$	$ \left(\begin{array}{c} \frac{1}{6} \\ \frac{1}{6} \\ \frac{1}{3} \\ \frac{1}{3} \\ 0 \end{array}\right) $	for $k = 1$ for $k = 2$ for $k = 3$ for $k = 4$ otherwise

Question 7: Geometric Distribution (15 points)

For X ~ Geo (0.20), find P (X > 5), P (2 < X \leq 6) and P (X > 5 | X < 8)

Question 8: Normal Distribution (10 points)

For X ~ N (4.3, 1.96), Use Normal Tables to find P (X \leq 5), P (2.8 \leq X \leq 5.5) and

P (X ≥ 4.3)

Assignment Submission

1. In paper format, all hand written. Write a report showing and explaining your solutions.

2. Full compliance with guidelines

3. Remember to submit the excel file through email before the deadline

4. Deadline for the submission is Thursday September 30th before class time (9:00 AM.)